The University of Chicago

The Structure of the Atom

Par I—Recent Work on the Structure of the Atom

Part II—The Changes of Mass and Weight Involved in the Formation of Complex Atoms

Part III—The Structure of Complex Atoms. The Hydrogen–Helium System

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE OGDEN GRADUATE SCHOOL OF SCIENCE IN CANDIDACY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

(DEPARTMENT OF CHEMISTRY)

By Ernest D. Wilson

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ESCHENBACH PRINTING COMPANY
EASTON, PA.

Part I

Recent Work on the Structure of the Atom

When Dalton[1] advanced his atomic theory of the constitution of matter, he thought of the atom as the ultimate material unit. The discovery of the phenomena of radioactivity, however, made it evident that this view was incorrect, and showed that the atom must be complex. The question of its structure has remained unsolved for a long time, and it is only very recently that there has been any experimental work upon which to base a theory. In this paper practically all of the important recent work bearing on this subject will be considered, and, wherever possible, the results due to the different investigators combined. As the results often seem to be contradictory, the difficulty of reaching any definite conclusion is great.

One of the first difficulties which arises in the attempt to develop an "atom model" is the fact that even at the present time we know nothing of the nature of positive electricity. The different characteristics of the negative electron have been known for some time, but the question of a positive electron is still open.

The first atom model was suggested by Lord Kelvin,[1] and consisted of concentric rings of rotating negative electrons in a sphere of homogeneous positive electricity the size of the atom. This model has come to be known as the Thomson atom, due to the fact that he developed it quite completely, and worked out in detail the number of electrons in the various rings necessary to give stable systems. The advantages of this construction lie in the relative simplicity of the mathematical calculation of the distribution and velocities of the electrons, as compared with the great difficulties involved in a satisfactory solution of such problems in connection with the later atom models. Thomson has shown that such an atom imitates to a large extent the properties of our known chemical atoms and explains why some are electropositive, others electronegative, and the variation of the chemical properties with the atomic weight.

One objection to this model, which also applies to the other models unless a rather questionable assumption is made, is that the atoms so formed would not be stable, for according to the electromagnetic theory, elec-

[1] "On Chemical Synthesis, from a New System of Chemical Philosophy," Manchester, 1808, pp. 211–6, 219–20.

trons in orbital motion must radiate energy, and hence at some time the atom would break up.

An objection that is even more important is that the positive spheres of Kelvin would have an electromagnetic inertia which would be negligible compared with that of even a single negative electron, leaving practically all of the mass of the atom unaccounted for on this theory.

The first theory of the constitution of the atom with any experimental work as a basis is due to Rutherford. He made use of the phenomenon of the scattering of alpha and beta particles in passing through matter.[2] The deflection is more marked for the beta than for the alpha particle, due to the smaller momentum of the former. There seems to be no doubt that the particles pass through atoms, and that their deflections are due to the intense electric field within the atom. Calculation shows that the distribution of the positive electricity assumed in the Thomson atom does not admit of sufficiently strong fields to deflect an alpha particle through a large angle, and the scattering of the alpha and beta rays in passing through thin sheets of metal had been attributed to a number of small scatterings. Geiger and Marsden,[3] working with gold foil 0.00004 cm. thick, found that about 1 in 20,000 alpha particles was deflected at an average angle of 90°. Also, Geiger[4] showed by the theory of probabilities that the most probable deflection was 0.87°, and that the chance of a 90° deflection was vanishingly small. In the theory of the Thomson atom the large deflections were considered as due to the accumulative effect of a number of small ones. The distribution of alpha particles deflected through large angles does not follow the probability curve. Also, after one collision the probability of the same ray suffering another in such a way as to give a larger deflection is very small.

Considering the deflections as due to a single encounter with an atom, Rutherford has obtained an expression for dm, the fraction of the total number of alpha particles which are deviated between p and $p + dp$.

$$dm = \pi/4\ ntb^2\ (\cot \phi/2\ \csc \phi/2)\ dp$$

where n is the number of atoms per unit volume, t the thickness of the metal, b the distance from the center to which an alpha particle would penetrate if shot directly at the atom, and p the perpendicular distance from the center of the atom to the extension of the line of path of the approaching alpha particle. Geiger[5] has shown this equation to be true between 30° and 150°.

Thomson considered the scattering as due to the accumulative effect of a number of small scatterings, and obtained the following expressions: The average deflection, θ_t, for a sheet of thickness t is $\theta_t = 3\pi b/8\ \sqrt{\pi n t}$. The probability p_1 that the deflection is greater than ϕ is $p_1 = e^{-\phi^2/\theta_t^2}$.

Rutherford shows that the probability p_2 for the same thing based on the theory of a single scattering is

$$p_2 = \frac{\pi}{4} b^2 nt \cot^2 \frac{\phi}{1}.$$

If $p_2 = 0.5$, $p_1 = 0.24$. If $p_2 = 0.1$, $p_1 = 0.0004$.

Thus the probability on Rutherford's assumption is greater than that on Thomson's. Both of these theories are developed on the assumption that the forces between the particles are electrical, and follow the inverse square law. Darwin has shown that this is the only law of force which is consistent with the facts.*

These considerations would seem to indicate that the large deflections actually found must be due to a single encounter with an atom, and in order to obtain the necessary strength of field, the positive charge would have to be concentrated at a small point instead of being evenly distributed throughout the entire volume of the atom, as in the theory of Thomson. The negative electrons vibrate around this positive nucleus, forming a kind of miniature solar system. This is the type of atom first suggested by Nagaoka.[6] From the measured deflections, Rutherford estimated that the charge on the nucleus was approximately equal to one-half the atomic weight of the element times the electronic charge. The large deflections are due to the alpha particle passing very close to the nucleus.

With beta rays the effects are slightly different, for since the force is attractive they increase speed on approaching the atom. By ordinary electrodynamics this involves a loss of energy by radiation, and an increase in apparent mass. Darwin showed that if the beta particle passed very close to the nucleus it would describe a spiral, and eventually fall in. This might explain the disappearance of swift beta particles in their passage through matter.

The case of the passage of the alpha particles through hydrogen is of particular interest,[7] for, since the alpha particle is heavier than the hydrogen atom, the recoil due to the close approach of the atom and the particle should be very large. It was shown that, as a result of a collision, the hydrogen atom should attain a velocity of 1.6 times, and hence a range of about four times that of the alpha particle itself. Marsden[8] actually found hydrogen atoms with a range of about 90 cm. in hydrogen in which the alpha particles had a range of only 20 cm.

From the data on the scattering of the alpha rays in passing through a

* NOTE.—A recent paper by Hicks takes into account the magnetic forces also, and shows that their effect may be of the same order of magnitude as that of the electrostatic forces only. Any theory which is complete must take account of both. Calculations based on only one would seem to be of doubtful reliability. See Hicks, *Phil. Mag.*, Jan., 1915.

gold leaf, it is possible to calculate an upper limit for the radius of the nucleus of the gold atom.

Ne = charge on the nucleus (positive).
R = the radius of the sphere of electric action.
Ne = the value of the negative charge surrounding the nucleus.
x = electric force at a distance r from the center.
v = potential force at a distance r from center.

Then $x = \text{Ne}\left(\frac{1}{r^2} - \frac{r}{\text{R}^3}\right)$ and $v = \text{Ne}\left(\frac{1}{r} - \frac{3}{2\text{R}} + \frac{r}{2\text{R}^3}\right)$.

m = mass of alpha particle.
u = velocity of alpha particle.
E = charge on the alpha particle.

Let the alpha particle be shot directly at the atom, being brought to rest at a distance b from the center. Then

$$^1/_2\, mu^2 = \text{NeE}\,(1/b - 3/\text{R} + b^2/2\text{R}^3).$$

If N is assumed to be 100, which cannot be very far from the correct value for gold, the distance b for an alpha particle of velocity 2.09×10^9 cm. per second is found to be 3.4×10^{-12} cm. This gives a maximum value for the radius of the nucleus of the gold atom.

Darwin[8] has made a similar calculation for the hydrogen atom and obtains the value 1.7×10^{-13} cm. for the diameter. This is smaller than the diameter of the negative electron, which is ordinarily given as 2.0×10^{-13} cm. The question arises as to whether the mass of this positive nucleus is entirely electromagnetic, like that of the negative electron. The electrical mass of a charged body is $2e^2/3a$, where e is the charge, and a the radius. Using this formula the radius of the positive nucleus of hydrogen comes out to be 1/1830 that of the negative electron. Rutherford[9] suggests that it is probable that the hydrogen nucleus is the long sought positive electron.

Rutherford has shown that it is impossible to account for the high speed of expulsion of some of the alpha and beta particles if they come from a ring of atomic radius. They must come from a point very close to the center, which suggests that they are shot from the nucleus of the atom.

The present theory of the constitution of the atom is based on the facts given above. Each atom of matter is supposed to be made up of a positively charged nucleus around which rotate the negative electrons. In the heavier atoms there are negative electrons in the nucleus also. The nucleus is the seat of practically all of the mass of the atom, for the negative electrons contribute very little mass. Barkla,[10] from his work on the passage of X-rays through matter, suggested that the charge on the nucleus is about 1/2 Ae, where A is the atomic weight of the element. Van den Broek,[11] and later Bohr,[12] suggested that the number of units of

charge is Ne where N is the number of the element when the elements are arranged in order of increasing atomic weight, or what is now called the atomic number.

Moseley's[13] work on the X-ray spectra of the elements is very important also, in helping to give an insight into the constitution of the atom. Working along the line first suggested by Laue,[14] and Bragg;[15] Moseley, and Darwin have developed an experimental method for determining the X-ray spectra of the elements by reflection from crystal surfaces. When an element is used as the anticathode in an X-ray tube, it emits a characteristic radiation of a frequency roughly 1000 times as great as that of the visible light waves. Each element is characterized by two different radiations which have been called the K radiation and the L radiation.[16] The K radiation is composed of two lines which Moseley has called the α line and the β line. These are the ones used the most by him for purposes of calculation. The L radiation is not nearly as penetrating as the K radiation, and usually consists of about five lines.

Moseley recorded the spectra photographically, instead of using an electroscope for a detector as did Bragg, and he examined he X-ray spectra of the elements from aluminium to gold. He found a very remarkable relationship between the frequencies of the lines of the various elements. In going from one element to the next higher in atomic weight there was a shift of the two lines of the K series toward the violet. That is, there is a change in the frequency of the spectrum lines with a change of the element. Moreover there is a very simple relationship between these frequencies. Thus considering the α line, the frequency is expressed by the formula

$$\nu = K(N - 1)^2$$

in which K is a constant, and N is a number which increases by one in passing from one element to the next higher in atomic weight. If 13 is chosen for the value of N for aluminium, and a corresponding value of K, N turns out to tbe the atomic number of the elements. Thus N determines the X-ray spectrum of any element. Moseley finds that between aluminium and gold the order of the elements according to N is the same as that of the atomic weights except where the latter would put the element out of place in the periodic table. Moseley finds only three unknown elements between aluminium and gold.

He has shown that neoytterbium and lutecium exist, but that Urbain's celtium is a mixture. According to his system thulium I and thulium II of Welsbach exist, but not thulium III. A homolog of Mn between Mo and Ru remains to be found as does another element between Os and W.

Moseley concludes that the number N is the charge on the nucleus of an atom, and that this charge increases one step at a time from one ele-

ment to the next. According to this view it is perfectly possible to have two elements of very different atomic weight that have the same X-ray spectrum, providing only that they have the same nuclear charge.

Such elements would also be identical chemically, and so far as is known at the present time would have identical spark and arc spectra.

This seems to be borne out by the fact that Aston,[17] working with J. J. Thomson, found two neons with atomic weights 20 and 22, which are otherwise identical, and have the same spectrum. He was only able to separate them by diffusion methods, which depend on the atomic weight and hence upon the density. Also Russell and Ross,[18] and Exner and Haschek,[19] working with a strong ionium solution, could not obtain any spectrum except that of thorium, of which ionium is an isotope.

This theory of the dependence of the chemical and physical properties of the elements on the nuclear charge is supported by recent work on the radioactive elements. The position of all these elements in the periodic table was an unsolved problem until it was shown that two or more elements could occupy the same place in the table. It has been supposed for some time that certain of the radio-elements were inseparable by any known chemical means. Fajans, Soddy, and others make this a general property of these elements, and treat each as the chemical analog of one or the other of the known elements. Two elements which occupy the same space in the periodic table and are inseparable by ordinary chemical means have been called "Isotopes" by Soddy.

The rule advanced by Soddy,[20] Fajans,[21] and others, that the expulsion of an alpha particle causes the element to shift its position in the periodic table two places to the left, and to decrease in atomic weight by four units is also in accord with the theory if we consider that the alpha particle comes from the nucleus of the atom. Similarly the expulsion of a beta particle, which is a negative electron, would cause the element to shift its position in the table one place to the right, without any change of mass. In this case the positive charge on the nucleus is increased by one unit.

In radioactive changes the expulsion of a beta particle, which evidently is shot out of the nucleus, as is shown by its extremely high velocity, causes the element to move one group to the right in the periodic table, which means an increase of one in the atomic number. From this it seems evident that this gives a proof, at least for the radioactive elements, and probably in the case of all lower atomic weight elements, that an increase of one in the atomic number means an increase of one in the positive charge on the nucleus. It seems probable, therefore, that the number of positive charges on the nucleus is equal to the atomic number.

TABLE I.

Element.	Radiation.	Valence.	Atomic no.	Atomic wt.
Ur I	α	6	92	238
UrX_1	β	4	90	234
UrX_2	β	5	91	234
Ur II	α	6	92	234
Io	α	4	90	230
Ra	α	2	88	226
Ra Em	α	0	86	222
Ra A	α	6	84	218
Ra B	β	4	82	214
Ra C	$\beta\ \alpha$	5	83	214
Ra C_1 Ra C_2[1]	$\alpha\ \beta$	6 3	84 81	214 210
Ra D	β	4	82	210
Ra E	β	5	83	210
Ra F	α	6	84	210
Pb	...	4	82	(206) 207

Element.[2]	At. wt.	At. no.	Element.	At. wt.	At. no.
Ur I	238	92	Ur X_1	234	90
Ur II	234		Io	230	
Ra A	218	84	Ra B	214	82
Ra F	210		Ra D	210	
			Pb	206	

[1] According to recent work by Miss Meitner there seems to be considerable doubt as to the existence of RaC_2.

[2] This table gives only the isotopes of the radium series.

The work of Rutherford and Andrade[22] serves to further confirm these views. The wave length of the soft γ rays from Ra B was determined, and also the X-ray spectrum of lead, with a view to determine the atomic numbers of these and the other radioactive elements. Ra B and lead were found to have the same atomic number, 82, as they gave the same X-ray spectrum. From Table I it is seen that there are several groups of elements having the same atomic number but with different atomic weights. These are tabulated at the bottom of Table I.

It will be noted that the atomic weight of lead, if its source is the uranium series of elements, should be 206.18, using the latest value of the atomic weight of uranium as 238.18. The atomic weight of lead has recently been determined by Richards,[23] who obtained the value 207.15 for ordinary lead, not radioactive. In analyzing the lead from a large number of radioactive minerals, mostly uranium, he obtained values varying from 206.4 to the ordinary value, 207.15. Soddy,[24] working with thorium minerals, obtained the value 208, which would be expected if the end of the thorium series is also lead. More recently Soddy[25] has started a more extensive investigation, and has obtained about 80 grams of lead

from thorium minerals. He finds that the lead has a higher density than the ordinary lead. This is to be expected if the atomic volumes of the isotopes are the same. Richards found no difference between the spectrum of the lead of low atomic weight and the ordinary lead, but Soddy claimed to have found at least one line in the new material which was not given by the old. In some work as yet unpublished, done by Aronberg working with Gale and Harkins no difference was found between the spectrum of some lead from Carnotite and the ordinary lead, although the Zeeman effect was also investigated. In this investigation a 21 foot concave grating was used, and the wave lengths could be measured to 0.001 of an Ångstrom.

Lindemann* shows by means of simple thermodynamical reasoning that two elements of different atomic weight must differ either in their chemical or physical properties. Since Soddy has shown that the isotopic forms of lead have the same atomic volumes, and also, of course, the same chemical properties, it follows that the forces between the atoms, and therefor the vapor pressures and the melting points, must vary, and that Soddy's lead from thorite should have a melting point 1.54 degrees higher than ordinary lead. Lindemann concludes that the forces of attraction and repulsion between the atoms have their origin in the nucleus, while, as is generally considered, the chemical properties and the radius of the atom are conditioned by the external electrons. In isotopes the forces of attraction and repulsion are proportional to the atomic weight, that is, probably to the number of positive particles. These forces are, however, not usually additive, but are so only in isotopes, so the nuclei of isotopes probably differ in their linear dimensions, but not at all, or very little, in the arrangement of the particles.

The fact that α particles are expelled in so many of the transformations seems to show that the nucleus is composed, in part at least, of helium atoms. The energy of the expulsion of the alpha particles can be accounted for by their passage through the intense electric field around the nucleus. The primary beta particles probably arise from a disturbance of the nucleus, which must be very complex.

The general facts which seem to be proved by all this work described above are, that the nucleus is a fundamental constant of matter, and that the charge on the nucleus determines the character of the element. The atomic weight is not so characteristic as the atomic number or the nuclear charge. The atomic weight is a complex function of the number and configuration of the electrons. Those properties of matter such as gravitation and radioactivity, which are entirely beyond our control by any chemical or physical agents, are functions of the nucleus.

* *Nature*, March 4, 1915.

Up to this point, beyond the fact that the atom is a sort of Saturnian system, nothing has been said as to the arrangement of the electrons in the atom, or the distribution of the forces. The first attempt to treat this problem as one of mechanics, and to give a definite picture of the atom, was made by Bohr.[26] In his calculations, Bohr has used an atom of the Rutherford type, and has combined with the classical mechanics Planck's quantum hypothesis. While Bohr's work has been severely criticized, the very remarkable results he was able to obtain with hydrogen alone make it worthy of careful consideration. Bohr's success lies only in the consideration of atoms with one vibrating electron. The following is a simplified form of his analysis:

Let m = mass of the electron.
$-e$ = charge of the electron.
M = mass of nucleus.
E = charge on nucleus.
a = radius of ring of rotation of electrons.
ω = frequency of revolution of electrons.
Then, $2\pi\omega$ = angular velocity.

The kinetic energy of an electron can be expressed in two ways:

$$\tfrac{1}{2}\, m(2\pi\omega a)^2 \quad \text{or} \quad \tfrac{1}{2}\, e\mathrm{E}/a.$$

The work necessary to remove an electron from its orbit to a position of rest at ∞ is:

$$e\mathrm{E}/a - \tfrac{1}{2}\, m(2\pi\omega a)^2 = \tfrac{1}{2}\,(e\mathrm{E}/a) = \mathrm{W}.$$

Then

$$2a = e\mathrm{E}/\mathrm{W} \quad \text{and} \quad \omega = \frac{\sqrt{2}}{\pi}\,\frac{\mathrm{W}^{3/2}}{e\mathrm{E}\,\sqrt{m}}.$$

According to Newtonian mechanics, the energy W should go on increasing as energy is given out by radiation, until the orbit gets smaller and smaller and the electron falls into the nucleus; that is, "a" would decrease and ω increase. Here Bohr introduces the quantum hypothesis. He assumes that the angular momentum of the electron is constant, and equal to $\tau h/2\pi$, where τ is an integer, and h is Planck's constant; that is, the angular momentum of the electron in its orbit is

$$2\pi m\omega a^2 = \mathrm{W}/\pi\omega = \tau h/2\pi.$$

This prevents continuous variations of W, a, and ω.

Then,

$$\mathrm{W} = \frac{2\pi^2 m e^2 \mathrm{E}^2}{\tau^2 h^2} \qquad 2a = \frac{\tau^2 h^2}{2\pi^2 m e \mathrm{E}}.$$

Much of the criticism of Bohr's work is directed at this point in his analysis. The idea of an electron undergoing accelerated motion without radiating energy is difficult to accept. There is also difficulty in ob-

taining any satisfactory physical picture of the process by which light is emitted when the electron changes from one steady state of vibration to the next, as described below.

In a neutral hydrogen atom τ is equal to 1. If this value is substituted in the second equation, the value for the diameter of the hydrogen atom is obtained as 1.1×10^{-8} cm., which is of the right order of magnitude.

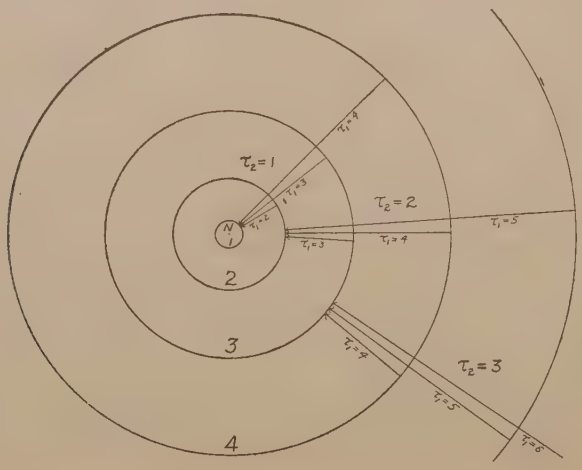
The electron radiates energy only when it changes from one steady state of vibration to another, and then one quantum of energy is released; that is, for a sudden shrinkage from orbit of τ_2 to τ_1, there must be a loss of energy $W = h\nu$ where ν is the frequency of the radiation.

$$h\nu = \delta W = W_{\tau_2} = W_{\tau_1} = \frac{2\pi^2 me^2 E^2}{h^2}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)$$

$$\nu = \frac{2\pi^2 me^2 E^2}{h^3}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right).$$

In the case of hydrogen, $E = e$. We can then calculate the value of the constant $2\pi^2 me^2 E^2/h^3$ which is equal to 3.26×10^{15}. The well-known Balmer formula for the series of lines in the hyrogen spectrum is $\nu = K(1/\tau_2^2 - 1/\tau_1^2)$ in which K as determined by experiment is 3.29 times 10^{15}. This practical identity of Bohr's calculated value of the Rydberg constant and the experimental value is probably the greatest triumph of Bohr's work.

If the value 1 is assigned to τ_2, and a series of values, 1, 2, 3, etc., given to τ_1, the frequencies of a series of lines in the ultraviolet are determined. This series was not known at the time of Bohr's first work, but has since been found by Lyman of Harvard. (Not published.) The physical picture obtained of the production of this series of spectrum lines is as follows:



N represents the nucleus of the atom. The rings 1, 2, and 3 correspond to the orbits of the electron in the various steady states of motion. When an electron falls from one steady state to the next one of smaller radius of vibration, one quantum of energy is liberated. In the above spectral series, all the lines are formed by electrons falling from the second ring and beyond, all the way to the first ring. The first line in the series is due to an electron falling from the second to the first ring; the second line to an electron falling from the third to the first ring, and so on.

If $\tau_2 = 2$, and a series of values be assigned to τ_1, the ordinary Balmer series for hydrogen results. For $\tau_2 = 3$, there results an infra-red series predicted by Ritz, and later discovered by Paschen. This model does not account for the Pickering series of lines which is ordinarily attributed to hydrogen, but Bohr shows that this series is accounted for by the helium atom.

Very recently another confirmation of Bohr's theory has been given by Evans[27] in his work on the spectra of hydrogen and helium. Bohr's formula, when modified so as to take account of the mass of the nucleus, is

$$\nu = \frac{2\pi^2 e^2 \mathrm{E}^2 \mathrm{M} m}{h^3(m + \mathrm{M})}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right).$$

This makes a slight change in the value of the constant in passing from hydrogen to helium, and the ordinary Balmer series for hydrogen, which according to Bohr's original work could come from either of the two elements, is found to be slightly different for the one than for the other. Thus it became very important to investigate carefully the spectra of these two elements, and see if this series could be detected in helium, and whether the slight differences just spoken of existed. Evans was able to observe the first few members of this series, and the measured values of the lines are very close to the theoretical.

Bohr[28] has also shown that if the principle of relativity is introduced, his formula takes the following form, which accounts for some of the extremely small errors found by Evans.

$$\nu = \frac{2\pi^2 e^2 \mathrm{E}^2 m \mathrm{M}}{h^3(m + \mathrm{M})}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)\left[1 + \frac{\pi^2 e^2 \mathrm{E}^2}{c^2 h^2}\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right)\right].$$

Helium is considered to have a charge on its nucleus of $2e$, or $\mathrm{E} = 2e$. The atom with one positive charge has one vibrating electron. The following formula results for helium:

$$\nu = \mathrm{K}\left(\frac{1}{(\tau_2/2)^2} - \frac{1}{(\tau_1/2)^2}\right)$$

The physical interpretation is the same as for hydrogen. For various values of τ_2, the following series result:

$\tau_2 = 1$—Extreme ultraviolet; not known.

$\tau_2 = 2$—Extreme ultraviolet; not known.

$\tau_2 = 3$—Two series, as τ_1 is odd or even. (The lines of the two series alternate.) These series were observed by Fowler in mixtures if hydrogen and helium, but had been attributed to hydrogen.

$\tau_2 = 4$—The lines of two series, alternating as τ_1 is odd or even. The first of these is the ordinary Balmer series, which evidently can come from either hydrogen or helium. The second is a series which was observed by Pickering in the star ζ-Puppis, and was attributed to hydrogen.

In the work of Evans which was mentioned, there is a further confirmation of Bohr's theory. By carefully adjusting conditions he was able to obtain the Pickering series of lines from absolutely pure helium, which should be the source of them, according to Bohr.

Balmer's series has never been observed in the laboratory beyond $\tau_2 = 12$, while in stellar spectra it extends to $\tau_2 = 33$. Therefore in vacuum tubes no hydrogen atoms exist of greater diameter than corresponds to $\tau_2 = 12$, or $2a = 1.6 \times 10^{-6}$ cm. For $\tau_2 = 33$, $2a = 1.2 \times 10^{-5}$ cm. Therefore, according to Bohr's theory, there are in the stars hydrogen atoms 1000 times the diameter of those on the earth.

Jeans[31] points out that in the above work the value of M is supposed to be very great in comparison with m. If this is not true, the value of Rydberg's constant is given by

$$K = \frac{2\pi^2 e^2 E^2}{h^3} \frac{mM}{(m + M)}.$$

If M refers to the value for hydrogen, 2M is the value for helium. Then $K_H : K_{He} = \frac{4}{M + {}^1/_2 m} : \frac{1}{m + M}$. From the best observed value of the ratio K_H/K_{He}, M/m is given as 1836 ± 12, which is in close agreement with the experimental value.

The above calculation would seem to be inconsistent with the idea of the atom as developed by Rutherford, for he states that practically all of the mass of the atom lies in the nucleus. In that case, the value of M for helium would not be *twice* but *four* times the value for hydrogen, and the helium nucleus would consist of four positive and two negative electrons.

Bohr extends his calculations to the lithium atom, and in considering only one vibrating electron obtains good results. In all of his work, however, when more than one electron is considered, his results are not correct. One of the serious objections to Bohr's theory is that he has been unable to explain the ordinary spectrum of hydrogen. Nicholson[22] has extended Bohr's calculations to every possible mode of vibration in attempting to secure a formula or formulas, giving the lines of the ordinary

hydrogen spectrum, but has found that they can be accounted for by no possible vibration.

Perhaps the most fundamental objection to Bohr's work lies in the fact that he has combined two basically different kinds of mathematics in working out this theory. It might be possible under such conditions to obtain results which are absolutely incorrect, for the two are contradictory.

Nicholson claims to have proven both by the classical mechanics and by Bohr's mechanics that coplanar, concentric rings of vibrating electrons are unstable. That is, if there are to be two or more rings of electrons in an atom, they cannot lie in the same plane, which would make Bohr's theory untenable. This presents difficulties in still another way. If we consider valence as due to certain electrons which are ordinarily considered as being near the outside of the atom, these electrons would either have to be in an outer ring by themselves, or else have some peculiar properties different from the other electrons in the ring. J. J. Thomsen not only believes in the existence of more than one ring of electrons, but is some work yet unpublished, states that he has actually counted the number of rings in certain atoms. This claim of Nicholson's would seem to be wrong, but it may be very true, as he says, that a large number of the vibrations in such a system are unsteady, and would result in the expulsion of an electron.

Bohr has not had better success in accounting for a large part of the spectrum of helium than he had in the case of hydrogen. Helium has six Balmer series which are not explained. Bohr considered only the first electron in his calculations on this element, and Nicholson thought that possibly the rest of the spectrum might be due to the other electron. He therefore made the necessary calculations, but could obtain no other series. Two of the series of lines calculated for helium lie in the ultraviolet, and are not knowm. Lyman of Harvard, as a result of his investigations, states that helium has no Schumann region spectrum.

It is well to remember that Bohr attempts no physical picture or cause of the change of an electron from one steady state of vibration to the next. In his theory of spectra, the energy which goes into the spectrum is atomic energy. This would be a serious objection if it were not for the fact that it is very easy to think of it in a slightly different way. The energy which it is necessary to apply to hydrogen to give these spectral lines may be used to remove an electron from one of the inner rings of vibration to one farther out from the nucleus. This is equivalent to increasing the atomic energy, and the increase, which came from an outside source, is given out as monochromatic radiation when the electron falls toward the nucleus.

Of the work on the structure of the atom none is more interesting than that of Nicholson, a number of his predictions from a theoretical standpoint having been confirmed in a most spectacular way. His work is also of extreme interest to chemists, since it deals with elements which have not as yet been discovered on the earth. For a considerable time it has been known that there are at least two important elements which are recognized by their spectrum, but which have not as yet been discovered on earth. A number of spectral lines of unknown origin were known to be given by the corona of the sun, and these were attributed to an element coronium, while the lines of unknown origin emitted by the nebulae were supposed to be due to an element which was given the name nebulium. The chief line of its spectrum is the line λ5007. Nicholson was able, by the assumption that certain lines belong to the spectrum of nebulium, to calculate the wave lengths of all of the other lines but two, for which he was altogether unable to account. Just at the time when Nicholson made his calculation, Wolf of Heidelberg was engaged in the study of these same lines in the spectra of the nebulae, and he found that certain lines, which had formerly been attributed to nebulium, acted differently from the others. He found, for example, that in the ring nebula in Lyra, discovered by Darquier in 1779, certain of these lines were emitted by the interior, and others by its outer part. The remarkable part of this discovery was that the two lines which were thus shown to have different origin from the true lines of nebulium, were just the two lines which Nicholson was unable to connect theoretically with the spectrum of nebulium. Another of Nicholson's predictions, which was confirmed in a remarkable manner, was that of the existence of a new nebulium line of wave length 4352.9. On photographing the spectrum, Wright, of the Lick observatory, found this line, and on looking over his older photographs, he found the line on a plate taken several years before the prediction of its existence was made, but the line was so weak that it had escaped observation.

In an extremely long series of papers Nicholson[30] has arrived at a very comprehensive theory of spectra, and has applied it to such problems as cosmic evolution. It gives a picture of operations in vast nebulae, many light years in extent, in connection with atomic and subatomic structure; changes occupying milleniums of time are discussed in connection with those occurring in a fraction of a second. Its very comprehensiveness causes some skepticism, but the remarkable results he has obtained would seem, in part at least, to justify the theory.

Nicholson also uses an atom of the Rutherford type, in which the electrons are vibrating in orbits around the positive nucleus. All of Nicholson's work is, however, based on calculations made by the classical mechanics. The quantum hypothesis is not introduced to obtain any

of the results, but in the course of his work, Nicholson shows how the results seem to be related to this hypothesis.

One of the interesting points of this theory is that the energy which goes into the spectrum is secured from the outside, and is not, as originally in the theory of Bohr, atomic energy. This would seem to be much more probable.

The electrons are considered as moving in a steady state, in a ring around the nucleus. Outside forces acting on these cause them to take up a vibration perpendicular to the plane of the ring. There are several modes of vibration, depending on the number of electrons in the ring. The strongest vibration would have a frequency equal to the frequency of the electrons in the ring. This is expressed by $q = \omega$, or $q/\omega = 1$. In vibrations of class zero the entire ring vibrates as a whole, always keeping parallel to its original position. The second class of vibration consists in the ring vibrating in halves. That is, there are two nodes and two crests in the wave which travels around the ring. It is evident that there are as many classes of vibrations as there are electrons in the atom. The vibrations of the higher classes would not be expected to be strong, and Schott has shown that the vibrations of a class higher than three would not ordinarily be strong enough to see. The mathematical analysis is largely the same as that developed by J. J. Thomson in his work on his atomic model, and will not be given completely here. Only the results which are used directly will be reproduced.

e = charge on an electron.
a = radius of the ring.
ν = no. charges in nucleus.
m = mass of an electron.
q/ω = frequency of revolution.

The equation for the period of vibration which Nicholson obtains is

$$mq^2 = e^2/8a^3\ (8\nu + P_k + P_o)$$

where

$$P_k = \Sigma^{n-1}\ (cos 2K\ s\pi/n\ csc^3\ s\pi/n)$$

and

$$P_o = \Sigma^{n-1}\ (1\ .\ csc^3\ s\pi/n).$$

There may be as many values of the period as there are of P_k, which is as many as the number of electrons. k is called the class of the vibration.

The first atom which Nicholson considered was one in which the number of positive charges in the nucleus is four. That is, $\nu = 4$. This atom he has called nebulium. First consider the neutral atom, in which there are four negative electrons. The period equation may be written as

$$ma^3q^2 = e^2\ [\nu + (P_k - P_o)/8],\ k = 0, 1, 2, \text{etc.}$$

If ω = the angular velocity of the ring, the force of an electron towards the center is $ma\omega^2$. The radial attraction of the positive nucleus is ve^2/a^2. The combined action of the other electrons gives a radial repulsion of $e^2S_n/4a^2$ where $S_n = \Sigma^{n-1} \csc s\pi/n$. Therefore,

$$ma\omega^2 = e^2/a^2\ (v - S_n/4) \quad \text{or} \quad e^2/ma^3 = \omega^2\ (v - S_n/4)$$

is the equation for the steady state of the neutral atom. The period equations of this system then become

$$q^2/\omega^2 = \frac{16}{15-2\sqrt{2}}, \quad k = 0$$

$$= \quad 1 \quad , \quad k = 1 = 3$$

$$q^2/\omega^2 = \frac{8}{8+\sqrt{2}} \ , \quad k = 2$$

or

$$q/\omega = (1.146533, \ 1, \ 0.849778)$$

Correcting for a stationary observer, and introducing negative values of k,

$$q/\omega = (1.146533, \ 1.150222, \quad 2, \quad 2.849778, \quad 4)$$
$$k = 0 \quad k = -2 \quad k = 1 \quad k = 2, \quad k = 3$$

In wave lengths (C = velocity of light) and considering a vibration in the plane where $q = \omega$, which we would expect to be strong,

$$\lambda = 2\pi C/\omega \ (1, \ 0.872194, \ 0.86939).$$

The first value is for $q = \omega$, the second for $k = 0$, the third for $k = -2$.

The chief nebular line is $\lambda = 5006.9$. If this is chosen as the fundamental line, then $(5006.9)(0.872194) = 4367.0$ should be the next line of this set. A strong line is known at 4363.4. Also $(5006.9)(0.86939) = 4352.9$ should be the next. At the time of this work, no line was known at 4352.9 but it was afterward found by Wright[31] of the Lick Observatory, and measured as 4352.3. Nicholson made similar calculations for this atom with 2, 3, 5, and 6 electrons. The results are given below:

		$q = \omega$.	$k = 0$.	$k = 1$.	$k = 2$.	$k = -2$.	$k = 3$.
$n = 6$	$\lambda = 2\pi C/\omega \times$			1	0.81934		(0.6666)
	$\lambda =$			4959.*a*	4063.2		....
	Observed.....			4959	4059.0		
$n = 5$	$\lambda = 2\pi C/\omega \times$	1	0.80991			0.81573	
	$\lambda =$	4743.0	3838.9*a*			3869.*a*	
	Observed.....	4740.0	3835.8			3869.	
$n = 3$	$\lambda = 2\pi C/\omega \times$	1	0.9250				
	$\lambda =$	4026.8*a*	3724.8				
	Observed.....	4026.8	3729.0				
$n = 2$	$\lambda = 2\pi C/\pi \times$	1	0.96826				
	$\lambda =$	3967.6*a*	3841.5				
	Observed.....	3967.6	3835.8				

"*a*" placed after a wave length shows that this one was assumed for that particular series.

The first method of attack used by Nicholson is briefly this, as is seen from the previous work. Making use of the theoretical analysis, he obtains the ratios of the wave lengths of what might be called a series, but not their absolute values. If a value for one of them is assumed, the rest are given. The agreement in some cases is far from good, and later it was found necessary to consider certain of the lines as due to a different source than given here. It is not clear as to why in the series for $n = 5$ for instance, the line for $k = -2$ should appear, and not the one for $k = 2$, or for the vibrations which should be still stronger, $k = 1$.

It will be seen that there is an error of about 4 A° in a large number of these calculated wave lengths. Nicholson shows that, except in the case of the vibration of Class O, this can be accounted for by the fact that the velocity of the electron is not small compared with that of light, or at least is not small enough so that it can be neglected without error. He calculates that the error due to this cannot be greater than about one part in a thousand, which amounts to about 4 A° with the wave lengths considered.

Bromwich pointed out to Nicholson that in the case of the vibration of Class O, the mass of the electron and the nucleus did not cancel out, but that a correction term of the form

$$(1 + nm/M)$$

should be introduced into the period equation. If H is the value of M for a hydrogen atom, then

$$m/H = 0.00054 \text{ approximately.}$$

This makes it possible to calculate the atomic weight of nebulium, which gives the value 1.3.

Using this method of ratios, Nicholson has calculated the wave lengths of lines for systems of $\nu = 2$, 3, 5, and 6. For $\nu = 2$ and 3, the lines were found by Wolfe in the nebulae. $\nu = 5$ was found in the solar corona, where Nicholson accounted for 21 out of 27 lines found.

For the systems $2e$, $4e$, $5e$, and $6e$, calculations have been made for the atomic weight by the method described above. From the values obtained, it can be shown that the atomic weights of these simple ring systems are proportional to the squares of the number of charges on their nuclei. For the entire series, the results are as follows:

Element	Pr.H	...	...	Nu	Pf	Arc
Atomic no	$1e$	$2e$	$3e$	$4e$	$5e$	$6e$
Atomic wt	0.082	0.327	0.736	1.31	2.1	2.9
Ratio	1^2	2^2	3^2	4^2	5^2	6^2

Here is a relation involving N^2 as in Moseley's work.

Up to this point, Nicholson used his method of ratios, and thus calculated the wave lengths of the most of the unknown lines in the nebulae

and the solar corona, and succeeded in securing approximate values for the atomic weights. No connection has as yet been shown to exist between the systems of different nuclear charge. This relation is brought out later. Also, no relation has been shown between the principal frequency of vibration of the systems of the same nuclear charge, with varying numbers of electrons. This relation is shown by the consideration of the energy of the systems involved.

As Nicholson points out, it is impossible to know the absolute energy of the atom,[1] but changes of energy may be used. By a very simple analysis Nicholson arrives at the following equation of energy of the atom, in which the term D is equal to the energy in a standard configuration:

$$^1/_2\, mnv^2 = ne^2/a\ (\nu - {}^1/_4\, S_n) + D$$

By the condition of steady motion,

$$ma^3\omega^2 = e^2\ (\nu - {}^1/_4\, S_n) = mav^2.$$

Now, $\omega = 2\pi C/\lambda'$, where λ' is the principal wave length. This gives

$$mna^2\omega^2 \cdot \frac{2\pi}{\omega} = \frac{mn}{C}\left\{\frac{e^2}{m}\ (\nu - {}^1/_4\, S_n)\ 2\pi C\right\}^{2/3}\lambda^{1/3}.$$

Therefore it is seen that the ratio of energy to frequency is proportional to $n(\nu - {}^1/_4\, S_n)^{2/3}\lambda^{1/3}$, and for convenience this will be called the atomic energy, E. Then $E = n(\nu - {}^1/_4\, S_n)^{2/3}\lambda^{1/3}$.

Now the calculation of E for the various atoms of protofluorine where $\nu = 5$, gives the following result:

	$n = 5.$	$n = 4.$	$n = 3.$	$n = 2.$
E.....	187.04	164.6	134.7	97.4

If 7.482 is chosen as the unit of energy, the number of units for the different systems is as follows:

	$n = 5.$	$n = 4.$	$n = 3.$	$n = 2.$	$n = 1.$	$n = 0.$
	25	22	18	13	7	0
Differences.....	3	4	5	6	7	
Units per electron.....	5	5.5	6	6.5	7	0

It is now possible to calculate the wave length of the line of principal frequency for the system $n = 2$. For this system it is seen that there are 6.5 units of E per electron, therefore

$$E = 2\ (6.5 \times 7.482) = 2\ (5 - {}^1/_4\, S_2)\lambda^{1/3} \qquad (S_2 = 1.000).$$

From this $\lambda = 5073$. A weak line is known at this point.

If the energy of one of these systems is decreased by radiation by certain discrete amounts, we would expect a series of lines of some sort to be the result. That is, a series of spectrum lines might emanate from atoms

[1] Equations have been developed from the standpoint of both the electromagnetic theory and the theory of relativity, which give the total energy of an atom in terms of its mass, but it is of course, not certain that these equations are valid.

whose internal angular momenta have run down by discrete amounts from a standard. In the case of protofluorine with two electrons, the loss of energy would be expected to be large enough so that the series could be observed. The wave lengths of the lines of this series may be expressed by the formula

$$\lambda = (97.107/2 - 1.223r)^3/(4.75)^2.$$

That is, the energy E is being decreased by an amount 2.2446 each time. The term r takes successively the values 1, 2, 3, etc. The series is as follows:

r	0	1	2	3	4	5	6
Calculated	5073	4725	4400	4086.5	3788	3506	3238
Observed	5073	4725	4400	4087.4	..	3505	..

All these are weak lines. It may be pointed out that this method of calculating a series is very similar to Bohr's method where he decreases the internal energy of the atom by discrete amounts to give the members of the series. Bohr has given some sort of an idea as to how this takes place, while Nicholson merely states that the energy is lost by radiation. Such vibrations as give rise to this type of series are vibrations in the plane of the ring, as in Bohr's model. It will be noted later that Nicholson makes considerable use of this method, even discarding his original identifications of some of the nebulium and protofluorine lines, and putting them into series of this type.

If Q equals the number of quanta of energy per electron, choosing as a unit the value $h/5$, where h is Planck's unit, a still more useful equation is obtained,

$$Q = E/n = \frac{\lambda^{1/3}(\nu - 1/4 S_n)^{2/3}}{0.06235}.$$

For the atom of protofluorine where $\nu = 5$, the various values of Q are as follows:

n	5	4	3	2
Q	600	658.5	718	778.5

These values of Q can easily be shown to be in the ratio of 10 : 11 : 12 : 13 : 14.

In the case of Pf just dealt with, the value of Q may be expressed by a function of the following type:

$$Q = A + Bn + Cn^2$$

where A, B, and C are constants. If such a formula is to be applied to other atoms than Pf, these terms may not be constants, but functions of the nucleus. Since the function E/n has been shown to be harmonic, it may be supposed that the function E/ν would be of the same general type, that is:

$$E/\nu = \alpha + \beta\nu + \gamma\nu^2,$$

where $v = n$, a neutral atom. Therefore the function E/n for any atom is of the form

$$E/n = A + Bv + Cv^2 - n(D + Ev) + Gn_2.$$

Since E in all cases is very closely a multiple of Planck's unit, the constants A to G will be close to whole numbers. The divergencies might be due to the rotation of the nucleus of the atom as a whole, at the expense of the angular momentum of the system, which otherwise might be exactly a multiple of $h/2\pi$. Nicholson calculates the values of these constants from the case of Pf, and then uses this method for the recalculation of the lines of the sytem $4e$, nebulium. He finds that it is necessary to change the source of a number of the lines from that given them by his first method. More of them are now thought of as coming from vibrations which fall into the type of series where the cube roots of the wave lengths differ by constant amounts, as in the series calculated for the system $v = 5$, $n = 2$. The value of Q for such series can be found from a formula of the type used in the series just referred to:

$$Q = (Q_n/18r),$$

where r is variable, taking successively the values 1, 2, 3, etc.

The law relating the number of quanta of positive and negative systems is:

$$(E/n)_{v-m} + (E/n)_{v+m} = 2(E/n)$$

A series might arise from negatively charged systems, from the energy disturbances due to the repulsion of negative electrons by the system. Such series might be expected to be quite strong. A neutral system would be unaffected by electrons, only becoming ionized by virtue of its own unstable vibrations.

Some very important relationships may be brought out by the consideration of the relations between the energy of the various neutral systems of different nuclear charge. If the principal wave length of the system is known, it is possible to calculate the value of E/n from the formula

$$E/n = \frac{\lambda^{1/3}(v - 1/4\, S_n)^{2/3}}{(0.06235)}.$$

The values of E/n for the systems $v = 5$, 4, and 2 are 600, 576.08, and 374.04, respectively. These are three terms of the following harmonic sequence:

v	5	4	3	2	1
E/n	600	576.08	500.76	374.04	195.92
Difference	23.92	75.32	126.72	178.12	
Second difference		54.40	54.40	54.40	

As stated before, it might be expected that these values of E/v, which are obviously the same as E/n, could be expressed by the formula

$$E/v = \alpha + \beta v + \gamma v^2.$$

The following three equations can be solved for the constants:

$$600.00 = \alpha + 5\beta + 25\gamma \quad (1)$$

$$576.08 = \alpha + 4\beta + 16\gamma \quad (2)$$

$$500.76 = \alpha + 3\beta + 9\gamma \quad (3)$$

and $\alpha = -32$, $\beta = 254.4$, $\gamma = -25.6$. Therefore

$$Q = E/\nu = 254.4\nu - 25.6\nu^2 - 32.$$

If the values of E/ν for the systems $6e$, $7e$, $8e$, $9e$ and $10e$ are calculated the following results are obtained:

ν	6	7	8	9	10
E/ν	572.52	493.64	363.36	181.68	—51.40

The fact that for $\nu = 10$ the energy is negative points to the fact that there are only 9 simple ring systems possible. If the first one is left out of consideration, and hydrogen is not put in the periodic table, the number of simple ring systems is the same as the number of groups in the first two rows of the periodic table. It will be shown later that the simple ring system $1e$ is very closely related to hydrogen so that leaving them both out of consideration at this point has some justification, particularly as the place of hydrogen in the table is very uncertain.

Knowing the value of E/ν for the various systems, it is possible to find their principal frequencies from the following formula:

$$\lambda = \left(\frac{E\ 0.06235}{\nu}\right)^3 \cdot \frac{1}{(\nu - {}^1/_4 S_n)^2}$$

ν	6	7	8	9
λ	2613.57	1323.12	431.02	45.15

All these are outside of the range of observation, but it will be shown later that the first of these exists in the nebulae, and it has been given the name arconium. It will also be shown that the system $1e$ exists, and it has been given the name protohydrogen. Its principal wave length is calculated as 1823.55, which is exactly one-half the limit of the Balmer formula for hydrogen.

The system $2e$ is present in the nebulae, and in *Nova Persei*. The atom with nucleus $3e$ is also present in the nebulae, but its lines are not as strong as those of $2e$. The system $4e$ is nebulium and gives rise to the strongest nebular lines. Protofluorine, $5e$, has not been found in the nebulae, but is one of the main constituents of the solar corona. As just stated, $6e$, arconium, and the system $7e$ are found in the nebulae.

The remarkable agreement of these theories of Nicholson's with the facts would seem to indicate that there is here an extremely broad foundation for a system on which the building up of the chemical elements may rest. The relations between the energy of these systems seems to indicate that the principle of the constancy of angular momentum may be the

physical basis of Planck's theory and also the basis of all the possible arrangements of electrical charges into the form of ordinary matter.

One of the most important and interesting pieces of work which Nicholson has done is in connection with the spectra of the Wolf-Rayet stars. These stars are considered by the astronomers as the earliest type known. They are regarded as being some sort of evolution product of the nebulae. The only ordinary elements which are present are hydrogen and possibly helium, although the last is not well represented unless, as Bohr suggests,* the Pickering series is due to this element. Nicholson shows that a number of Balmer series exist in these stars, and identifies them in a remarkable manner with the constants of the simple ring system called nebulium.

If the lines of these stars are examined they are found to contain a number of lines in the ratio of 5/4. Thus:

5285/4228 = 1.2500	5693/4555 = 1.2499
5813/4652 = 1.2495	5593/4473 = 1.2503

If the Balmer formula is considered for one special case the following relationship is found:

$$\lambda = \lambda_0 m^2/m^2 - 1$$

For $m = 2$ and 4 the following results:

$$\lambda_2 = 4/3\ \lambda_0 \qquad \lambda_4 = 16/15\ \lambda_0 \qquad \lambda_2/\lambda_4 = 5/4$$

This suggests that Balmer series exist in the Wolf-Rayet stars, and a simple calculation gives the following:

$$\lambda = (4104,\ 3963.5)\ m^2/m^2 - 1$$

A further examination of the lines leads to the following four series:

$$\lambda = (4104,\ \text{or}\ 3963.5)\left(\frac{m^2}{m^2 - 1}\ \text{or}\ \frac{m^2}{m^2 - 0.3286}\right)$$

It was found in the previous work that the three main vibrations of the neutral nebulium atom had frequencies in the following ratio:

$$q^2/\omega^2 = 1,\ 1.3145,\ 0.8496$$

Now $0.3286 = 1.3145/4$. This shows that these series are in some way related to the nebulium atom. It seems probable that another series or rather two series might exist in which the term 0.3286 was replaced by one-fourth of the other ratio for the third line of the neutral atom.

$$0.8496/4 = 0.2124.$$

The formula would then be

$$\lambda = (4104,\ 3963.5)\ m^2/m^2 - 0.2124.$$

Lines calculated for $m = 2$, and 3 give lines which are known in these stars, thus confirming the theory.

* This suggestion is confirmed by the work of Evans.

In general then, the formula for the Balmer series in the Wolf-Rayet stars may be written as

$$\lambda = (4104,\ 3963.5)\left(\frac{m^2}{m^2 - q^2/4\omega^2}\right).$$

A search for another limiting frequency gives the value 5254. We now have nine Balmer series which may be written as

$$\lambda = (4254,\ 4104,\ 3953.5)\left(\frac{m^2}{m^2 - q^2/4\omega^2} \text{ or } \frac{m^2}{m^2 - 1}\right).$$

A consideration of the limiting wave lengths gives still another way of expressing these formulae. If they are written in wave numbers their differences are seen to be constant.

$$10^3/\lambda_0 = 25227,\ 24366.5,\ 23506$$

with the constant difference 860.5. Therefore $\lambda_0^{-1} = A + Bn$ where n takes integral values. Substituting values for this and rearranging it, the following formula for the Balmar series in the Wolf-Rayet stars is obtained:

$$\lambda = \frac{2\lambda_0}{1 + nd}\left(\frac{m^2}{m^2 - q^2/4\omega^2} \text{ or } \frac{m^2}{m^2 - 1}\right)$$

in which $\lambda_0 = 5007$, the principal wave length of the neutral atom of nebulium, n takes successive integral values, and $d = 0.08609$ and is not arbitrary, but is calculated from data involved in the consideration of the neutral nebulium atom.

The above equation is one of the most remarkable and important points of Nicholson's work, for in it he succeeds in bringing together the simple ring systems which are not known on the earth, and our ordinary elements.

It is evident that a change in the term n means a change of some sort in the atom. It is also evident that it must be of a character that the force between the component parts does not change. It seems inevitable that the change must consist in a change in the nucleus, which does not alter its charge. It is significant that the simple ring systems seem to be incapable of giving rise to Balmer series, and the only logical conclusions is that they depend on the intimate structure of the nucleus. The elements which give rise to Balmer series may then be looked upon as the evolution products of the simple ring system on which the value of λ_0 depends. This is considering λ_0 as belonging to any simple ring system and not to nebulium alone.

Nicholson shows that the simple ring system $1e$ can have only one vibration frequency, $q = \omega$. This has already been calculated as $\lambda = 1823.35$, which is exactly one-half of the limiting wave length of the Balmer series for hydrogen. From the formula

$$\lambda = \frac{2\lambda_0}{1 + nd}\left(\frac{m^2}{m^2 - 4}\right),$$

it is seen that, to derive the hydrogen Balmer formula, n must be equal to 0. That is, hydrogen would appear to be the first evolution product of the system $1e$, protohydrogen.

It should be noted that Nicholson has in a rather round-about way succeeded in evaluating the Rydberg constant even more closely than did Bohr. He has not, however, attempted to give any physical picture of the mechanism of the production of the Balmer series.

While up to the present time the evolution products of the other systems are not known in general, it is likely that they may be some of our terrestrial elements. The lines due to a number of the evolution products of nebulium are shown in the Wolf-Rayet stars.

No work has yet been published connecting helium with the simple ring systems, although from the fact that it appears in the nebulae and the stars about the same time as does hydrogen, it seemed likely that it is one of the early evolution products of one of the systems. In some work as yet unpublished Nicholson has succeeded in showing that helium is the evolution product of the system $2e$.

From the progressive changes of the spectra of the stars, and other evidence which points to their age, it seems likely that all of our common elements are more complex forms of matter than the simple ring systems and hydrogen, and are built up from them. In such spectra the elements make their appearance in the order of their atomic weights.

According to Nicholson's idea, then, our ordinary terrestrial elements are evolution products of the simple ring systems found in the nebulae and the stars. That is, they have nuclei which are complex, containing both positive and negative electrons. This is in good accord with other facts but it does not agree with Rutherford's idea that the nucleus of the hydrogen atom may be the positive electron. If the astronomical evidence is admitted, it may be safely assumed that in the novae and nebulae, where the temperature is supposed by Nicholson to be very high, the complex atoms are unstable, breaking up into simple ones, or if the process is looked at from the other point of view, as these hot bodies cool the simple ring systems condense, becoming more complex and giving rise to the elements known on the earth.

The only very important papers on atomic structure which have not been considered are those of Stark. His work has been omitted on account of lack of space, and because he has not as yet been able to obtain from his results any very definite picture of the structure of the atom. Another reason is that this work is already available in book form[31] which is not true of the material presented in this paper. Stark's greatest discovery is that of the electrical Zeeman effect, or the Stark effect, which

is the decomposition of the spectral lines by means of a static electrical charge. Up to the present time this effect, which has been made to give a much greater separation of the components of the lines than has been obtained by the Zeeman effect, has been obtained only in the spectra from canal or positive rays. However this is probably due to the fact that this is the only method which has been found for securing the high field strengths which are essential. In Stark's work he has used a potential fall of from ten to seventy-four thousand volts per centimeter.

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Part II

The Changes of Mass and Weight Involved in the Formation of Complex Atoms

In the study of the important question of the structure and composition of the elements, it might seem that a consideration of the relations existing between the atomic weights should give results of the greatest value. Unfortunately, however, the first suggestions presented to explain the relations which probably exist were given in such a form, and were based upon such extremely inaccurate values for the atomic weights that a very considerable prejudice has been developed against similar hypotheses.

The first important hypothesis in regard to atomic weight relations appeared in two anonymous papers in the Annals of Philosophy for 1815 and 1816, just one hundred years ago. These papers were known to have been written by Prout, whose ideas as they were presented received the vigorous support of Thomson, considered in England as the leading chemical authority of his day; and many years later, from 1840 to 1860, they were very strongly advocated by Dumas, who made a large number of atomic weight determinations during this period. Very many other chemists, among them Gmelin, Erdmann, and Marchand, were also numbered among Prout's supporters. On the other hand, Stas, who in the beginning tried to aid Dumas in the revival of Prout's hypothesis, afterward designated it as a pure fiction, and Berzelius at all times adhered to the view that the exact atomic weights could not be determined except by experiment.

The prejudice which existed a few years ago against Prout's idea is well shown by a quotation from von Meyer's History of Chemistry, printed in 1906.

"During the period in which Davy and Gay-Lussac were carrying on their brilliant work, and before the star of Berzelius had attained to its full luster, a literary chemical event occurred which made a profound impression upon nearly all the chemists of that day, *viz.*, the advancement of Prout's hypothesis. This was one of the factors which materially depreciated the atomic doctrine in the eyes of many eminent investigators. On account of its influence upon the further development of the atomic theory this hypothesis must be discussed here, although it but seldom happens that an idea from which important theoretical conceptions sprang, originated in so faulty a manner as it did."

Prout's work was not, as the above quotation infers, entirely "literary," for he made a large number of experimental determinations for use in his calculations of the specific gravity of the various elements, which he as-

sumed to exist in the gaseous form. His experiments were, according to his own statements, somewhat crude, but he also made use of the more accurate data obtained by Gay-Lussac, and his work was based upon the volume relations of gases as discovered by the French investigator.

Exactly the form in which the numerical part of Prout's hypothesis should be expressed in terms of modern atomic weights, it is difficult to say, but the principal point is that his atomic weights, which, however, are not comparable with those now used, were expressed in whole numbers, as given below in two columns taken from his table:

TABLE I.—PROUT'S TABLE OF THE MORE ACCURATELY DETERMINED ATOMIC WEIGHTS.

Element.	Sp. gr.	Atomic weight, 2 vols. of hydrogen being 1.
H	1	1
C	6	6
N	14	14
P	14	14
O	16	8
S	16	16
Ca	20	20
Na	24	24
Fe	28	28
Zn	32	32
Cl	36	36
K	40	40
Ba	70	70
I	124	124

The atomic weights thus given by Prout are within a few units of the modern values in the case of the univalent atoms and for nitrogen; but the values given for the atoms of higher valence, with the exception of nitrogen, are approximately half the present values. This would mean that according to Prout's system, since the atomic weights he gives are whole numbers, the atomic weights of the present system should be divisible by two for the atoms of higher valence, which is equivalent to the use of the hydrogen molecule instead of the atom as a unit. In this connection it may be noticed that his atomic weights are taken on the basis of "2 volumes of hydrogen being 1."

Thus, from a numerical standpoint, Prout's hypothesis does not seem to mean what is usually supposed. Expressed in terms of the composition of what he considered to be complex atoms, it is given below in his own words:

"If the views we have endeavored to advance be correct, we may also consider the *πρώτη ὑλη* of the ancients to be realized in hydrogen, an opinion by the way, not altogether new. If we actually consider this to be the case, and further consider the specific gravities of bodies in their gaseous state to represent the number of volumes condensed into one; or, in other words, the number of the absolute weight of a single volume of the first matter which they contain, which is extremely probable, multiples in weight must also indicate multiples in volume, and *vice versa;* and the specific gravities, or abso-

lute weights of all bodies in the gaseous state, must be multiples of the specific gravity or absolute weight of the first matter, because all bodies in a gaseous state which unite with one another, unite with reference to their volume."

While it is true that Prout had at the time when he presented it, no real foundation for his ideas, more accurate work, while it proved his system to be invalid from a purely numerical standpoint, at the same time established the fact that the atomic weights of the lighter elements, on the hydrogen basis, are much closer to whole numbers than would be likely to result from any entirely accidental method of distribution. Thus the deviations of the lighter elements are small, as will be seen by the following table:

Element.	At. wt. H = 1.	Deviation from a whole number.
He	3.97	0.03
Li	6.89	0.11
Be	9.03	0.03
B	10.91	0.09
C	11.91	0.09
N	13.90	0.10
O	15.88	0.12
F	18.85	0.15

The average of these deviations is 0.09 unit, while the theoretical deviation on the basis that the values for the atomic weights are entirely accidental, is 0.25 unit. If the first seventeen elements are used in the calculation, the average deviation is found to be 0.15 unit, while the result obtained for twenty-five elements is 0.21. The more complete table, designated as Table II, gives these deviations, which are seen to be negative in almost every case, the exceptions being magnesium, silicon, and chlorine. The exclusion of beryllium from consideration in this connection is due to the fact that its atomic weight is not known with sufficient accuracy, and neon is not taken into account, since its positive variation may be explained by the discovery by Thomson and by Aston that neon is a mixture of two isotopes of atomic weights twenty and twenty-two.

Not only is the variation from a whole number a negative number, but in addition its numerical value is nearly constant, the average value for the 21 elements being 0.77%, while the six elements from boron to sodium show values of 0.77, 0.77, 0.70, 0.77, 0.77, and 0.77%. The deviation is therefore not a periodic, but a constant one. If, then, a modification of Prout's hypothesis that the elements are built up of hydrogen atoms as units is to be taken as a working basis, it becomes important to find a cause for the decrease in weight which would result from the formation of a complex atom from a number of hydrogen atoms. The regularity in the effect suggests that, in general, this decrease in weight is probably due to some common cause, though the exceptional cases of

TABLE II.—DEVIATIONS OF THE ATOMIC WEIGHTS FROM WHOLE NUMBERS.

	At. wt. H = 1.	Diff. from whole number.	Per cent. variation or the packing effect.	Possible per cent. variation.	At. wt. O = 16.	Diff. from whole number.	Per cent. variation from whole no.	Prob. error in at. wts.
H[1]	1.000	....	....	....	1.0078	+0.0078	0.78	0.0002
He[2]	3.97	—0.03	—0.77	12.5	4.00	0.00	0.00	0.01
Li	6.89	—0.11	—1.62	7.1	6.94	—0.06	—0.86	0.01
Be	9.03	(+0.03)	....	5.5	9.1	+0.1	(+1.11)	0.05
B	10.91	—0.09	—0.77	4.5	11.0	0.00	0.00	0.05
C[3]	11.91	—0.09	—0.77	4.2	12.00	0.00	0.00	0.005
N	13.90	—0.10	—0.70	3.6	14.01	+0.01	+0.07	0.005
O	15.88	—0.12	—0.77	3.1	16.00	0.00	0.00	0.00
F	18.85	—0.15	—0.77	2.6	19.00	0.0	0.0	0.05
Ne[3]	19.85	....	....	....	20.0	....	....	....
Na[4]	22.82	—0.18	—0.77	2.2	23.00	0.00	0.00	0.01
Mg	24.13	+0.13	+0.55	2.15	24.32	+0.32	+1.33	0.03
Al	26.89	—0.11	—0.40	1.85	27.1	+0.10	+0.37	0.1
Si	28.08	+0.08	+0.31	1.78	28.3	+0.30	+1.07	0.1
P[5]	30.78	—0.22	—0.71	1.61	21.02	+0.02	+0.06	0.01
S[3]	31.82	—0.18	—0.56	1.56	32.07	+0.07	+0.22	0.01
Cl	35.19	+0.19	+0.54	1.43	35.46	+0.46	+1.31	0.01
Ar	39.57	—0.43	—1.07	1.25	39.88	—0.12	—0.30	0.02
K	38.80	—0.20	—0.52	1.28	39.10	+0.10	+0.25	0.01
Ca	39.76	—0.24	—0.60	1.25	40.07	+0.07	+0.17	0.03
Sc	43.76	—0.24	—0.55	1.14	44.1	+0.10	+0.23	0.2
Ti	47.73	—0.27	—0.57	1.04	48.1	+0.10	+0.21	0.1
V	50.61	—0.39	—0.77	0.98	51.0	0.0	0.0	0.1
Cr	51.60	—0.40	—0.77	0.96	52.0	0.0	0.0	0.05
Mn	54.50	—0.50	—0.90	0.90	54.93	—0.07	—0.13	0.05
Fe	55.41	—0.59	—1.06	0.89	55.84	—0.16	—0.29	0.03
Co	58.51	—0.49	—0.83	0.85	58.97	—0.03	—0.05	0.02

Per cent. variation of 21 elements (omitting Be, Mg, Si, Cl), or the packing effect = 0.77%

Average devation of the atomic weights, H = 1, from whole numbers = 0.21

Theoretical deviation of atomic weights from whole numbers on the basis that the deviations are entirely accidental = 0.25

Average deviation of the atomic weights, H = 1, for the eight elements from helium to sodium = 0.11

Average deviation of the atomic weights, O = 16, when Mg, Si, and Cl are omitted = 0.05

Average deviation of the atomic weights, O = 16, for the eight elements from helium to sodium = 0.02

[1] W. A. Noyes ("A Text-book of Chemistry," p. 72) states that the atomic weight used for hydrogen, 1.0078, is probably not in error by so much as 1 part in 5000.

[2] Heuse (*Verh. deut. physik. Ges.*, **15**, 518 (1913)) obtained the value 4.002 as the result of 7 experiments.

[3] Leduc (*Compt. rend.*, **158**, 864 (1914)) gives the atomic weight of neon as 20.15 when hydrogen is taken as 1.0075. Leduc's value is not used, on account of the discovery of the complexity of neon as described in the text of the paper.

[4] Richards and Hoover (*Jour. Amer. Chem. Soc.*, **37**, 95 (1915)) determined the atomic weights of carbon as 12.005, and of sodium as 22.995, and in Vol. **37**, p. 108, they give the atomic weight of sulfur as 32.06.

[5] The atomic weight for phosphorus is taken as 31.02 from recent determinations made by Baxter (*Jour. Amer. Chem. Soc.*, **33**, 1657 (1912)).

magnesium, silicon, and chlorine, show that there is certainly some other complicating factor. The discovery of the reason for the deviation of the same kind in the case of neon, where it is due to its admixture with an isotope of higher atomic weight, suggests that it may not be impossible to find explanations for these three other exceptions. In order to have a term for the percentage decrease in weight, it may be well to call this the packing effect, or the percentage variation from the commonly assumed law of summation, that the mass of the atom is equal to the sum of the masses of its parts.

It has formerly seemed difficult to explain why the atomic weights referred to that of oxygen as 16 are so much closer to whole numbers than those referred to that of hydrogen as one, but, the explanation is a very simple one when the facts of the case are considered. The closeness of the atomic weights on the oxygen basis to whole numbers, is indeed extremely remarkable. Thus for the eight elements from helium to sodium the average deviation is only 0.02 unit, which is less than the average probable error in the atomic weight determinations. When twenty-one elements are taken from the table, omitting the exceptional cases of magnesium, silicon, and chlorine, the deviation averages only 0.05 unit, while if these are included, this is increased only to 0.09 unit. These results have been calculated without taking the sign into account. If the sign is considered the average deviation is reduced to 0.01 unit for the twenty-one elements. The probability that such values could be obtained by accident, is so slight as to be unworthy of consideration. If an oxygen atom is a structure built up from 16 hydrogen atoms, then the weight according to the law of summation should be 16 times 1.0078 or 16.125. The difference between 16.125 and 16.00 is the value of the packing effect, and if this effect were the same for all of the elements, except hydrogen, then the choice of a whole number at the atomic weight of any one of them, would, of necessity, cause all of the other atomic weights to be whole numbers. Though this is not quite true, it is seen that the packing effect for oxygen is 0.77%, which is the average of the packing effects for the other 21 elements considered. Therefore, those elements which have packing effects equal to that of oxygen will have whole numbers for their atomic weights, and since the other elements show nearly the same percentage effect, their atomic weights must also lie close to whole numbers.

According to this view, Prout's hypothesis from the purely numerical standpoint, is entirely invalid, but there still remains the problem of finding an explanation for three facts: First, that the atomic weights of the lighter elements on the hydrogen basis approximate whole numbers; second, that the deviations from whole numbers are negative; and third, that the deviations are practically constant in magnitude. Before con-

sidering any explanation of these facts it is of interest to consider the following extremely interesting comments upon this subject, as written by Marignac in 1860:

"We are then able to say of Prout's hypothesis that which we can say of the laws of Mariotte and Gay-Lussac relative to the variations of the volumes of gases. These laws long considered as absolute, have been found to be inexact when subjected to experiments of so precise a nature as those of M. Regnault, M. Magnus, etc. Nevertheless they will be always considered as expressing natural laws, either from the practical point of view, for they allow the change of volume of gases to be calculated in the majority of cases, with a sufficiently close approximation, or even from the theoretical point of view, for they most probably give the normal law of changes of volume, when allowance has been made for some perturbing influences which may be discovered later, and for which it may also be possible to calculate the effects. We may believe that the same is true with respect to Prout's law; if it is not strictly confirmed by experiment, it appears nevertheless to express the relation between simple bodies with sufficient accuracy for the practical calculations of the chemist, and perhaps also the normal relationship which ought to exist among these weights, when allowance is made for some perturbing causes, the research for which should exercise the capacity and imagination of chemists. Should we not, for example, quite in keeping with the fundamental principle of this law, that is to say, in admitting the hypothesis of the unity of matter, be able to make the following supposition, to which I attach no further importance than that of showing that we may be able to explain the discordance which exists between the experimental results and the direct consequences of this principle? May we not be able to suppose that the unknown cause (probably differing from the physical and chemical agents known to us), which has determined certain groupings of primordial matter so as to give birth to our simple chemical atoms, and to impress upon each of these groups a special character and peculiar properties, has been able at the same time to exercise an influence upon the manner in which these groups of atoms obey the law of universal attraction, so that the weight of each of them is not exactly the sum of the weights of the primordial atoms which constitute it?"

It has usually been assumed, and without any really logical basis for the assumption, that if a complex atom is made up by the union of simple atoms, the mass of the complex atom must be exactly equal to the masses of the simple atoms entering into its structure. Rutherford, from data on the scattering of α-rays in passing through gold leaf, has calculated an *upper* limit for the radius of the nucleus of a gold atom as 3.4×10^{-12} cm. The mass of this relatively heavy atom is, according to this calculation, practically all concentrated in this extremely small space, which is so small that it could no longer be expected that the mass of such a nucleus, if complex, would be equal to the sum of the masses of its component parts. In fact, since the electromagnetic fields of the electrons would be so extremely closely intermingled in the nucleus, it would seem more reasonable to suppose that the mass of the whole would not be equal to the sum of the masses of its parts. The deviation from the law of summation cannot be calculated on a theoretical basis, but it can easily be determined from the atomic weights, if the assumption is made that the heavier atoms are condensation products of the lightest

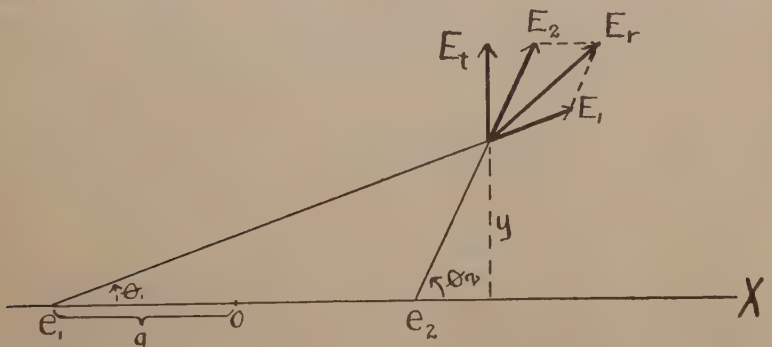
of the ordinary elements, that is of hydrogen. This deviation expressed in terms of the percentage change, is what has already been determined, and designated as the packing effect.

Since this packing effect represents a decrease in weight, the first problem which represents itself for determination is the *sign* of the effect which would result from the formation of the positively charged nucleus of an atom by the combining of positive and negative electrons into some form of structure. Richardson[1] suggests that the positive nucleus of an atom might be built up of positive electrons alone and still be stable if the law of force between them were

$$+ a/r^2 - b/r^{p_1} + c/r^{p_2}, \text{ where } p_2 > p_1 > 2.$$

Here the first term gives the usual law of force, the second causes the electrons when close together to attract each other, and the third expresses the repulsion which keeps them from joining together. It would, however, seem more simple to assume, what seems much more probable, that the nucleus is held together by the attraction of positive and negative electrons, both of which are assumed to be present in any complex nucleus.

Since, even when the mass is assumed to be entirely electromagnetic, there still remain two possibilities even for the simple case of hydrogen, first, that the hydrogen nucleus is the positive electron, and second, that it may be complex, it has seemed best to choose for the purpose of calculation the simplest system, which consists of one positive and one negative electron. The problem thus presented for solution is the determination of the sign and the magnitude of the change of mass which results when a positive and a negative electron are brought extremely close together.



Lorentz[2] speaks of this problem, but does not solve it either with respect to the sign or the magnitude of the effect. He does state, however, that if the electrons were to be brought into immediate contact, the total

[1] "The Electron Theory of Matter," p. 582.

[2] H. A. Lorentz, "The Theory of Electrons," 1909, pp. 47 and 48.

energy could not be found by addition, which may be considered as equivalent to the statement that the mass of a system made up in this way would not be the same as the sum of the masses of its parts. The fundamental equations used here as the basis of the calculation which follows, have been taken from the work of Lorentz.

The value of e, the charge on the electron, may be defined as

$$e = \int\int\int \rho d\tau,$$

where ρ is the volume density of the electricity, and $d\tau$ is an element of volume. For the purposes of the first part of the calculation, the electron may be considered as a point charge, but the values of the electromagnetic mass used later are given for the Lorentz form of electron, which takes the form of an oblate spheroid when in motion.

The space surrounding an electron must be considered as different from a space not adjacent to an electrical charge. If a charged particle is brought into this space it is acted upon by a force which varies from point to point, and has at every point in space a definite value and direction. This force is designated by $\mathbf{E}$, and is a vector point function. If the electron is in motion it acts as an electric current equal to eu, where u represents its velocity. The magnetic force due to this motion is easily seen to be a function of the current equivalent of the moving electron, and is also a vector, designated by $\mathbf{H}$. Then

$$\mathbf{H} = f(\mathbf{E}, u, \phi)$$

where ϕ is the angle between $\mathbf{E}$ and the direction of motion. The direction of $\mathbf{H}$ is perpendicular to the direction of u and is at the same time circular.

It is evident that the total energy of the system is a function of both the electric and the magnetic intensities. For the purposes of this calculation the mass of a system is considered as electromagnetic, and hence as a function of the energy of the system. Therefore it is necessary to use some function of both $\mathbf{E}$ and $\mathbf{H}$. This function is designated by $\mathbf{G}$ and is called the electromagnetic momentum. The derivation of the equations for $\mathbf{G}$ has been given by Lorentz, so here it will be sufficient to define it as

$$\mathbf{G} = [\mathbf{E}\,\mathbf{H}]/c$$

where $[\mathbf{E}\,\mathbf{H}]$ means the vector product of $\mathbf{E}$ and $\mathbf{H}$, and c is the velocity of light. From the expressions obtained for $\mathbf{G}$ it is easy to obtain those which represent the mass.

In the treatment which follows, only the longitudinal electromagnetic mass is considered, and terms containing u to a higher power than the first are disregarded, as they appear to be unimportant. The following general treatment, in which Heaviside units are used, gives an outline of the method:

For the field due to a system of charges

$$\frac{[\mathbf{EH}]}{c} = \frac{[(\Sigma_i \mathbf{E}_i)(\Sigma_j \mathbf{H}_j)]}{c} = \frac{\Sigma_i [\mathbf{E}_i \mathbf{H}_i]}{c} + \frac{\Sigma_{(ij)} [\mathbf{E}_i \mathbf{H}_j]}{c}$$

where the summation $\Sigma_{(ij)}$ is the vector product of each i with each j. The first summation gives the electromagnetic momentum which would be due to the particles if their fields did not overlap, and the second term, which is the important one here, gives the effect of the overlapping of the fields. This may be called the "mutual electromagnetic momentum" and designated by $\overline{\mathbf{G}}$.

For point charges

$$\mathbf{E}_1 = \frac{(1 - u^2)e_1}{4\pi r^2 (1 - u^2 \sin^2 \theta_1)^{3/2}}$$

at the point $P_{x, y, z}$. Let

$$(1 - u^2 \sin^2 \theta_1) = \beta_1^2$$

and

$$(1 - u^2) = k^2.$$

The transverse component of $\mathbf{E}$ due to the two particles 1 and 2 is

$$\mathbf{E}_t = \frac{k^2 e}{4\pi} \left\{ \frac{\sin \theta_2}{r^2 \beta_1^3} \pm \frac{\sin \theta_2}{r^2 \beta_2^3} \right\}$$

where the sign is *positive* if the charges have the same sign, and *negative* if they are of opposite sign. As only the longitudinal component of the vector $\mathbf{G}$ is desired, only the transverse component of $\mathbf{E}$ is needed.

$$\mathbf{H} = u/c \, \mathbf{E} \sin \phi$$

where ϕ = the angle between $\mathbf{E}$ and the direction of u. If $\mathbf{E}_t'$ is used, $\phi = 90°$. Therefore

$$\mathbf{H} = u/c \, (\mathbf{E}_1 \sin \theta_1 \pm \mathbf{E}_2 \sin \theta_2)$$

$$\mathbf{G}_L = \frac{[\mathbf{E}_t \mathbf{H}]}{c} = \frac{u}{c_2} (\mathbf{E}_1 \sin \theta_1 \pm \mathbf{E}_2 \sin \theta_2)(\mathbf{E}_1 \sin \theta_1 \pm \mathbf{E}_2 \sin \theta_2).$$

And

$$\overline{\mathbf{G}} = \pm \frac{2u}{c^2} \int \mathbf{E}_1 \mathbf{E}_2 \sin \theta_1 \sin \theta_2 \, d\tau$$

$$= \pm \frac{2u}{c^2} \frac{k^4 e^2}{(4\pi)^2} \int \frac{\sin \theta_1 \sin \theta_2}{r^2 r^2 \beta_1^3 \beta_2^3} \, d\tau.$$

Now

$$r^2 \beta^2 = r^2 - u^2 (r^2 \sin^2 \theta) \quad \text{and} \quad (r^2 \sin^2 \theta) = y^2.$$

Neglect all of the terms in u^2.

$$d\tau = 2\pi y dy dx.$$

Then

$$\overline{\mathbf{G}} = \frac{u k^4 e^2}{8 c^2 \pi^2} \cdot 2(2\pi) \int_0^\infty \int_0^\infty \frac{y^3 dy dx}{\sqrt{\{[(x - a)^2 + y^2][(x + a)^2 + y^2]\}^3}}$$

which is obtained by making use of the symmetry of the equation. Or

$$\overline{G} = \left(\frac{uk^4e^2}{8c^2\pi^2} \cdot 4\pi\right)\frac{1}{a}\int_0^\infty\int_0^\infty \frac{y^3dxdy}{\sqrt{\{[(x-1)^2+y^2][(x+1)^2+y^2]\}^3}}$$

$$= \frac{uk^4e^2}{2\pi c^2 a} J$$

where

$$J = \int_0^\infty\int_0^\infty \frac{y^3dxdy}{\sqrt{\{[(x-1)^2+y^2][(x+1)^2+y^2]\}^3}}$$

$$J = \frac{1}{2}\int_0^\infty dx \int_0^\infty \frac{udu}{[(\alpha+u)(\beta+u)]^{3/2}}$$

where $u = y^2$.

$$\alpha = (x-1)^2; \quad \beta = (x+1)^2; \quad \alpha - \beta = (-4x).$$

$$J = \frac{1}{2}\int_0^\infty dx \left[\frac{2[(\alpha+\beta)u + 2\alpha\beta]}{(\alpha-\beta)^2[(\alpha+u)(\beta+u)]^{1/2}}\right]_0^\infty$$

$$= \int_0^\infty \left[\frac{(\alpha+\beta)}{(\alpha-\beta)^2} - \frac{2\alpha\beta}{(\alpha-\beta)^2\sqrt{\alpha\beta}}\right]dx$$

$$= \int_0^\infty \frac{1}{16x^2}[\alpha+\beta-2\sqrt{\alpha\beta}]\,dx$$

$$= \int_0^\infty \frac{[\sqrt{\alpha}-\sqrt{\beta}]^2}{16x^2}\,dx$$

$$= \int_0^\infty \frac{[(x-1)-(x+1)]^2}{16x^2}\,dx$$

$$= \int_0^1 \frac{(-2x)^2}{16x^2}\,dx + \int_0^\infty \frac{(-2)^2}{16x^2}\,dx$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

$$\therefore \overline{G} = \frac{uk^4e^2}{4\pi ac^2}$$

The mass represented by this value of $\overline{G}$ is

$$\Delta m = \pm e^2/4\pi c^2 a.$$

Now the longitudinal mass, m_1 is

$$m_1 = e^2/6\pi c^2 R,$$

where R is the radius of the electron. By division

$$\Delta m/m_1 = 3R/2a,$$

where a is equal to one-half the distance apart of the electrons.

In the application of this last equation, R must be taken as the radius of the positive electron, since it is assumed that it is the seat of practically

all of the mass of the atom. For a decrease of mass of 1% in this simple case the distance apart of the positive and negative electrons would be, according to the equation, 300 times the radius of the positive electron. In order to produce a decrease of mass equal[1] to the average decrease of weight found for the 21 elements given in Table II, or 0.77%, the distance apart of the two electrons as calculated, would be 400 times the radius of the positive electron. This, however, does not give the resuls for any actual case which is known, and in general the nucleus of an atom must be more complex than this. In a more complex nucleus it is possible that the positive and negative electrons need not come so close together in order to give the same decrease of mass. It is evident that the calculation cannot be applied to any special atom until the mass of the positive electron is determined. If, as Rutherford seems to think probable, the positive nucleus of the hydrogen atom is the positive electron, then the most probable composition of the helium nucleus would be four positive electrons to two which are negative, and it would not seem improbable that in such a system the effect upon the mass of the positive electrons might be greater than in the simpler case used for the calculation, which would mean simply that the positive and negative electrons need not be so close together to produce the same effect on the mass. Whether this is true or not could not be determined without a knowledge of the structure of the helium nucleus. If, as Nicholson assumes, the hydrogen nucleus is complex, the decrease of mass in the formation of one helium atom from four of hydrogen, would be due to the closer packing of the positive and negative electrons in the helium nucleus.

Earlier in the paper it has been shown that the fact that the atomic weights on the oxygen basis are much closer to whole numbers than those on the hydrogen basis, is explained by what has been called the packing effect, or the change of mass involved in the formation of heavier atoms from hydrogen. The average of the packing effects for the elements considered, is 0.77%. This is also the value of this effect for oxygen, which happens to have been chosen as the fundamental element in the determination of atomic weights. If the number representing atomic weight of hydrogen, 1.0078, is decreased by this percentage amount, it becomes 1.0000, which is the fundamental unit in atomic weight determinations. The atomic weights of the twenty-five fundamental elements listed in Table II, are found, on the whole, to be very nearly products of this unit by a whole number. While the numerical unit of measurement does not change, the actual unit of mass, the mass of the hydrogen nucleus, varies

[1] From the electromagnetic theory the velocity of high speed electrons also exerts a perceptible influence upon the mass, but the magnitude of this effect has not as yet been determined for the case of the electrons in an atom.

slightly from atom to atom, and this variation causes the slight deviation of the atomic weights from whole numbers.

The opposite of the system here proposed would be, to suppose that the values of the atomic weights are wholly the result of accident. On this basis the probability that the atomic weights fall as close to whole numbers as they do, may be calculated. In such calculations oxygen is omitted, since its atomic weight is fixed as a whole number, and hydrogen is not used, since its atom contains only one hydrogen nucleus.

The first calculation made was that of the probability that *each* of the atomic weights should be as close as it is to a whole number. The data used are those of Table II. The chance that the atomic weight of nitrogen should entirely, by accident, deviate from a whole number by only 0.01 of a unit was determined by dividing the unit into the 200 divisions corresponding to the assumed accuracy (Landolt-Börnstein-Meyerhoffer, Tabellen) of 0.005. The greatest possible deviation would then be 100 divisions, while the actual deviation of 0.01 unit corresponds to 2 divisions. The probability is then one-fiftieth. The chance that any number of independent events should all happen is the product if the separate probabilities of their each happening. The probability calculated in this way is 2×10^{-22}, or

$$\frac{2}{10{,}000 \text{ billion billion}},$$

which indicates that there is practically no chance that the atomic weights are entirely the result of accident.

Another probability, which seems to be of more value in connection with the present problem, is that the *sum* of the deviations shall not exceed the sum actually found. This is of the form known as De Moivre's problem, and the method was used by Laplace[1] in calculating the probability that the sum of the inclinations of the orbits of the ten planets to the ecliptic is not greater than the value found at that time, 0.914187 of a right angle. The result obtained was 1.1×10^{-7}. The problem is stated in the following way: An urn contains $n + 1$ balls marked, respectively, 0, 1, 2, 3, ... n; a ball is drawn and replaced: required the probability that after i drawings the sum of the numbers drawn will be s. This probability is the coefficient of x^s in the expansion of

$$\frac{1}{(n+1)^i}(1 - x^{n+1})^i(1 - x)^{-i},$$

or the probability P is

[1] Laplace, "Oeuvres VII, Theorie Analytique des Probabilites," pp. 257–62.

$$P = \frac{1}{(n+1)^i}\left\{\frac{\lfloor i+s-1}{\lfloor i-1 \lfloor s} - \frac{i}{1}\frac{\lfloor i-1+s-n-1}{\lfloor i-1 \lfloor s-n-1} + \frac{i(i-1)}{1.2}\frac{\lfloor i-1+s-2n-2}{\lfloor i-1 \lfloor s-2n-2} \cdots\right\}.$$

In the case of the atomic weights P gives the probability that the sum of the deviations from whole numbers shall equal s, which is not what is desired. The result wanted is the probability that the sum of the errors shall be equal to or less than s, or the summation of the Ps from 0 to s. Now

$$\sum_{s=0}^{s=s} \frac{\lfloor i+s-1}{\lfloor i-1 \lfloor s} = \frac{\lfloor i+s}{\lfloor i \lfloor s}$$

So the desired probability, P′ is:

$$P' = \sum_{s=0}^{s=s} P = \frac{1}{(n+1)^i}\left\{\frac{\lfloor i+s}{\lfloor i \lfloor s} - \frac{i}{1}\frac{\lfloor i+s-n-1}{\lfloor i \lfloor s-n-1} + \frac{i(i-1)}{1.2}\frac{\lfloor i+s-2n-2}{\lfloor e \lfloor s-2n-2} - \frac{i(i-1)(i-2)}{1.2.3}\frac{\lfloor i+s-3n-3}{\lfloor i \lfloor s-2n-3} \cdots\right\}.$$

In solving this problem all of the first twenty-seven elements have been used with the exception of hydrogen and oxygen, and these *should* be omitted for the reasons given above. The errors in the determined values have been taken as they are given in Table II. The atomic weights used in the calculation are as follows:

He.....	4.002	Mg.....	24.32	Ca.....	40.07
Li.....	6.94	Al.....	27.1	Sc.....	44.1
Be.....	9.1	Si.....	28.3	Ti.....	48.1
B.....	11.0	P.....	31.02	V.....	51.0
C.....	12.005	S.....	32.06	Cr.....	52.0
N.....	14.01	Cl.....	35.46	Mn.....	54.93
F.....	19.00	A.....	39.88	Fe.....	55.84
Ne.....	20.15	K.....	39.10	Co.....	58.97
Na.....	22.995				

The average probable error as determined from Table I is 0.043 unit, which is equivalent to about 24 divisions for one unit, or 12 divisions for half a unit, which is the maximum possible deviation from a whole number. Since n is 12, $n + 1$ is taken as 13. The sum of the deviations from whole numbers is 2.342, which is equal to 56 of the divisions determined above, or $s = 56$. The number of elements, i, is 26. The probability calculated on this basis is 6.56×10^{-8}, or approximately

$$\frac{1}{15{,}000{,}000}.$$

It has been assumed in this paper that the cause of the deviations of neon, magnesium, silicon, and chlorine, which are exceptional in giving positive deviations from the atomic weights on the hydrogen basis, must be differ-

ent from that which gives the deviations of the other elements. The cause of the deviation of neon has been explained, but for the others it is unknown. In the calculations of the probabilities given above these elements have been included. It may be of interest to note that if these elements had been excluded the probability for the 21 remaining elements would have been found to be about

$$\frac{1}{7 \text{ billion}}.$$

It is an interesting coincidence that the probability above found for the 27 lighter elements is about 1×10^{-7}, while the probability determined by Laplace that the sum of the inclinations of the ten planets then known, to the ecliptic, should not be greater than the sum of the measured values, is almost the same, or 1.123×10^{-7}. In the second paper of this series it will be shown that the atomic weights not only approximate whole numbers, but that these whole numbers are in addition *certain* numbers which are determined by a special system, and which may be given accurately by an equation of the form

$$W = 2(n + n') + 1/2 + [(-1)^{n-1} \times 1/2].$$

The probability that the atomic weights should come so close to these *special* whole numbers is much less than that calculated above, so that the words of Laplace may be applied to the system presented here, as well as to the one he himself gives. That the atoms are built up of units of weight very close to one, and that therefore this modified form of Prout's hypothesis holds, "est indiquee avec une probabilite bien superieure a celle du plus grand nombre des faits historiques sur lesquels on ne se permet aucun doute."

The accepted atomic weights on the oxygen basis as now used are closer to whole numbers than those given by Ostwald in 1890.[1] Ostwald's numbers are all larger than the corresponding whole numbers, so the deviations were all positive. On the other hand, the present values show both positive and negative deviations. The fact that the small change of 0.77% from the oxygen to the hydrogen basis eliminates practically all of the tendency of the atomic weights to be near whole numbers, when as many as 27 elements are considered as in Table II, shows that the atomic weight of oxygen cannot be taken as very different from 16.00 without obscuring this relationship. Thus it has been shown that the probability that the atomic weights on the oxygen basis would come entirely by accident as close to whole numbers as they do, is 6.56×10^{-8}, or about

$$\frac{1}{15{,}000{,}000}.$$

[1] *Allgemeine Chemie*, 1, p. 126 (1890).

A change of only 0.77% from the oxygen basis causes an enormous increase in the probability that the atomic weights obtained in this way could be as close to whole numbers as they are, entirely by accident. Thus the chance that the sum of the deviations should come out as equal to, or less than, the sum actually found, is 0.105, or

$$\frac{1}{10}$$

As has been seen, there are 27 atomic weights distributed over 59 units of atomic weight. The greatest common divisor of the whole numbers corresponding to the atomic weights is one. The atomic weights are therefore such that numerically they seem to be built up from a unit of a mass of one, and the probability results seem to show that this unit of mass must be very close to 1.000, expressed to three decimal places. On the other hand, this unit of mass must be somewhat variable to give the atomic weights as they are, even although a part of the variation, in some cases, may be due to the inaccuracy with which the atomic weights are known. This leads either to the supposition (1) that the atoms are built up of some unknown elementary substance, of an atomic weight which is slightly variable, but is on the average extremely close to 1.000, and which does not in any case deviate very far from this value, or to the idea (2) which is presented in this paper, that the *nucleus* of a known element is the unit of structure. The atom of this known element has a mass which is close to that of the required unit, and it has been proved that the decrease of mass involved in the formation of a complex atom from hydrogen units is in accord with the electromagnetic theory. The adoption of the first hypothesis would involve much more complicated relations. It would necessitate the existence of another elementary substance with an atomic weight close to that of hydrogen, it would involve a cause for the increase of weight in the formation of some atoms, and a decrease in other cases, and it would also involve the existence of another unit to give the hydrogen atom.

In the second paper, which follows, still more evidence in favor of the theory that the other atoms are complex atoms built up from hydrogen units will be presented, and it will be shown that there is also an important secondary unit of structure.

The writer wishes to thank Professor A. C. Lunn, of the Department of Mathematics, for outlining for him the mathematical analysis of the determination of the packing effect.

Summary.

1. The atomic weights of the first 27 elements, beginning with helium, are not multiples of the atomic weight of hydrogen by a whole number, as they would be if Prout's original hypothesis in its numerical form

were true. This may be expressed by the statement that the atomic weights on the hydrogen basis are not whole numbers. However, when these atomic weights are examined critically it is found that they differ from the corresponding whole numbers by a nearly constant percentage difference, and that the deviation is negative in sign, with an average value of —0.77%.

2. This percentage difference has been called the packing effect, and it represents the decrease of weight, and presumably the decrease of mass, which must take place if the other atoms are complexes built up from hydrogen atoms. The regularity in this effect is very striking, the values for a number of the lighter atoms being as follows: He, —0.77; B, —0.77; C, —0.77; N, —0.70; O, —0.77; F, —0.77; and Na, —0.77%, while the average value for the first 27 elements is —0.77%.

3. The regularity of the packing effect gives an explanation of the well-known fact that the atomic weights on the oxygen basis are very close to whole numbers, while this is not true of the atomic weights on the hydrogen basis except in the case of the lightest elements from helium to oxygen. The atomic weight of hydrogen on the oxygen basis is 1.0078. If this were decreased by the value of the packing effect of 0.77%, it would become a whole number, 1.000. Then, if the other elements are built up from hydrogen atoms as units, all of the atoms which are formed with a packing effect of —0.77%, must have whole numbers for their atomic weights; thus the atomic weights of the elements listed in Section 2, above, must be whole numbers in six of the seven cases listed, He, B, C, O, F, and Na. The fixing of any one of these six atomic weights as whole numbers causes the other five to be whole numbers also. Thus the atomic weights referred to carbon as 12.00 would be the same as those referred to oxygen as 16.00. A variation of the atomic weight of an element on the oxygen basis from a whole number indicates that the packing effect for that element does not have the average value.

4. Recent work has shown that the nucleus of an atom must be extremely minute. Thus Rutherford gives the *upper* limit for the radius of the relatively large and complex gold atom as 3.4×10^{-12} cm., while Crehore, who proposes another theory of the structure of the atom, considers that none of the electrons have orbits of a greater radius than 10^{-12} cm. The high velocity with which the β-particles are shot out in radioactive transformations has been considered as evidence that these electrons must come from much closer to the center of the atom than the assumed radius of the atom. It therefore seems practically certain that the electrons and positively charged particles which make up the nucleus of a complex atom, are packed exceedingly closely together. As a result of this close packing, the electromagnetic fields of the charged particles must overlap to a considerable extent, which would mean that

the mass of the atom ought not to be equal to the sum of the masses of the individual particles from which it is built.

5. The closeness to which a positive and a negative electron would have to approach to give a decrease of mass equal to 0.77%, or the average value of the packing effect, is found by calculation to be to a distance of 400 times the radius of the positive electron. This case does not correspond to any element actually known, for the simplest of the atoms considered, helium, may be supposed to have a nucleus built up from four hydrogen nuclei and two negative electrons. However, the magnitude of the effect seems to be of the order which would be expected.

6. The probability for the first 27 elements, that the sum of the deviations of the atomic weights (on the oxygen basis from whole numbers) should by accident be as small as it is, is found to be one chance in fifteen million. On the other hand, a change of only 0.77% from the oxygen basis to that of hydrogen gives one chance in ten that the atomic weights should be as close to whole numbers as they are.

Part III

The Structure of Complex Atoms. The Hydrogen–Helium System

In Part II it has been shown that the atomic weight relations of the elements are such as to make it extremely probable that the atoms are complex structures built up from hydrogen atoms. It therefore becomes important to determine in what way the hydrogen atoms unite together to make up the complex. Rutherford proved that the α-particles which are shot out in the disintegration of the radioactive elements, have a mass of four units,[1] and that they give ordinary helium gas when they escape through the walls of a thin glass capillary tube in which the emanation is stored.[2] Fajans,[3] Soddy,[4] Russell,[5] von Hevesy,[6] and Fleck,[7] have found that, when a radioactive substance ejects an α-particle, the new substance has different properties, and a different valence from those of the parent material. The change is such that the new element lies two places to the left in the periodic table, and therefore has an atomic number which is two less than before the alpha disintegration. It has been found that uranium, for example, can lose eight α-particles in eight steps, and change into a form of lead. From this it is seen that the radioactive elements, which have high atomic weights, must, at least in part, be built up of α-particles, and therefore of helium atoms, with this difference, that while the α-particle is probably present as a whole in the complex atom, the nonnuclear electrons of the helium atom, undoubtedly rearrange themselves in the complex atom, so that the helium atoms as a *whole* do not preserve their identity.

Now that it has been proved that the atoms of high atomic weight are built up, in part at least, of helium atoms, the question arises as to whether the same relations hold for the lighter atoms which have not been found to give an appreciable alpha disintegration. If they do, then a change

[1] *Phil. Mag.*, [6] **28**, 552–72 (1914).

[2] Rutherford and Soddy, *Phil Mag.*, **3**, 582 (1902); 453 and 579 (1903); Ramsay and Soddy, *Nature*, p. 246 (1903); *Proc. Roy. Soc.*, **72**, 204 (1903); **73**, 346 (1904); Curie and Dewar, *Compt. rend.*, **138**, 190 (1904); Debierne, *Ibid.*, **141**, 383 (1905); Rutherford, *Phil. Mag.*, **17**, 281 (1909).

[3] *Physik. Z.*, **14**, 131–6 (1913).

[4] *Chem. News*, **107**, 97 (1913), and *Jahrb. Radioakt.*, **10**, 188 (1913).

[5] *Ibid.*, **107**, 49 (1913).

[6] *Physik. Z.*, **14**, 49 (1914).

[7] Fleck, *Trans. Chem. Soc.*, **103**, 381 and 1052 (1913).

of two places to the right in the periodic table, which is more accurately expressed as an increase of two in the atomic number, should increase the atomic weight by the weight of one helium atom, or by the number four. Since a change of two in the atomic number should increase the atomic weight by four, according to this theory, the average increase in the atomic weight per atomic number should be two. From this it might be expected that the tenth element would have an atomic weight equal to 20, and the twentieth element, an atomic weight of 40. That this is actually the case is seen, for neon, the tenth element has an atomic weight of 20, and calcium, the twentieth element, has a weight of 40. In order to investigate the question more in detail, a start may be made with helium, of an atomic number 2 and a weight of 4. The element of an atomic number four, should be heavier by the weight 4, or its atomic weight should be equal to eight. Above this the elements, if built up according to this helium system would have the weights:

Atomic number.	Atomic weight.	Group number.
6	12	4
8	16	6
10	20	0
12	24	2
14	28	4
16	32	6

where each step is made by adding the weight of one helium atom. The equation which represents the idea that the atomic weights of the lighter elements, belonging to even numbered groups, change in the same way as the elements in a radioactive series (namely, by an amount equal to four for a change of two groups in the periodic table), is

$$W = 2n,$$

where W is the atomic weight and n is the atomic number.

If a similar system is supposed to hold for the odd numbered elements, then beginning with lithium of an atomic weight seven, and an atomic number three, the atomic weights according to the simple helium system would be:

Atomic number.	Atomic weight.	Group.
3	7	1
5	11	3
7	15	5
9	19	7
11	23	1
13	27	3
15	31	5
17	35	7
19	39	1

It is thus seen that for the odd groups as well as the even, the increase

in the atomic weight is just that predicted for the addition of one helium atom for each step of two atomic numbers. The even and odd numbered elements are thus seen to belong to two different series. A single equation for both of these series may be easily written by introducing a term which disappears when n is even, and is effective when n is odd. If W is the atomic weight,[1]

$$W = 2n + \{1/2 + [(-1)^{n-1} \times 1/2]\}.$$

In Table I the atomic weights calculated according to this equation are given for the elements up to and including cobalt.

TABLE I.—A COMPARISON OF THE CALCULATED AND THE DETERMINED VALUES OF THE ATOMIC WEIGHTS.[2]

Element.	n.		Calculated.	Detd.	Diff.	Probable error in detn.
He	2		4	4.0	0	0.01
Li	3		7	6.94	+0.06	0.05
Be	4		8	9.1	—1.1 (= 1H)	0.05
B	5		11	11.0	0	0.05
C	6		12	12.00	0	0.005
N	7		15	14.01	+0.99 (= 1H)	0.005
O	18		16	16.00	0	...
F	9		19	19.0	0	0.05
Ne	10		20	20.0	0	...
Na	11		23	23.00	0	0.01
Mg	12		24	24.32	—0.32	0.03
Al	13		27	27.1	—0.1	0.1
Si	14		28	28.3	—0.3	0.1
P	15		31	31.04	—0.04	0.1
S	16		32	32.07	—0.7	0.01
Cl	17		35	35.46	—0.46	0.01
A	18		36	39.88	—3.88 (= 1He)	0.02
K	19		39	39.10	—0.10	0.01
Ca	20	n'	40	40.07	—0.07	0.07
Sc	21	1	44.1	44.1	—0.1	0.1
Ti	22	2	48	48.1	—0.1	0.1
V	23	2	51	51.0	0	0.1
Cr	24	2	52	52.0	0	0.05
Mn	25	2	55	54.93	+0.07	0.05
Fe	26	2	56	55.84	+0.16	0.03
Co	27	2	59	58.97	+0.03	0.02

It is interesting to note that of the 28 elements in this table, 13, or very

[1] Although it was not known to the writer at the time when this paper was written, it was found on looking up the subject that Rydberg, in an extremely important paper published in 1896 (*Z. anorg. Chem.*, **14**, 80) found from a study of atomic weight relations that the elements belong to two series corresponding to the two formulas $4n$ and $4n-1$, where n is a whole number. Thus from an empirical basis he derived the same relationships as are developed in this paper from an entirely different standpoint; that is by the application of the relations found between the elements in a single radioactive series to the elements of small atomic weight.

[2] For the final equation including n' see section 3 of the summary.

nearly half, have atomic weights which are divisible by 4, and that of all of the possible multiples of 4, only two are missing, *i. e.*, 2×4 and 9×4. Seemingly to make up for the omission of the 9×4, the 10×4 occurs twice. This may be represented as follows:

1×4 = He	8×4 = S
2×4 = missing, but represented by $(2 \times 4) + 1$	9×4 = missing, but replaced by 10×4 = A
3×4 = C	10×4 = Ca
4×4 = O	11×4 = Sc
5×4 = Ne	12×4 = Ti
6×4 = Mg	13×4 = Cr
7×4 = Si	14×4 = Fe

Of the atomic weights given in the table only one is divisible by 2, which is at the same time not divisible by four. Seven, or one-fourth of the atomic weights, are divisible by 3, though the threes are not evenly spaced like the fours; three are divisible by 5, and two of these, argon and calcium, have the same atomic weight. Five are divisible by 7, and two by 9, and every possible multiple of 16 appears. According to this the most important numbers are 4 and 3, which is in accord with the equation given for the atomic weights, 3 being an important secondary unit.

Of the twenty-six elements given in this table, it is found that the equation gives the atomic weights of nine, or more than a third, with no difference between the calculated and determined values, and for six other elements the difference is practically within the limits of error of the determinations. For the three elements, Be (+1.1), N (—0.99), and argon (—3.88), the differences in the first two cases are practically equal to the weight of a hydrogen atom, and for argon the difference, when allowance is made for a possible change of the packing effect, is the weight of a helium atom. The deviations of magnesium (0.32), silicon (0.3), and chlorine (0.46), are somewhat large, the largest deviations being that of chlorine, which is equal to 1.3% of its atomic weight. These deviations are also exceptional in that they are greater on the basis of oxygen as 16 than they are on the basis of hydrogen as 1.00.

If these six cases of deviation, three of which can be explained as due to a deviation in the number of hydrogen or helium units, are neglected, it is found that for the other twenty elements the equation gives the atomic weights with so great an accuracy that the average deviation is only 0.045 unit, which is practically equal to the average probable error in the experimentally determined values as given by Landolt-Börnstein.

It has been seen that for the first twenty elements the average increase in weight is 2.00, or exactly the same increase as is found for the uranium or the thorium radioactive series. For the heavier elements the increase is somewhat more rapid. The increments are tabulated in Table II.

TABLE II.—THE CHANGE IN THE ATOMIC WEIGHT WITH THE ATOMIC NUMBER.

Change of atomic number.	Final element.	Atomic wt.	Average increment.
0–10	Ne	20	2.0
10–20	Ca	40.07	2.007
20–30	Zn	65.37	2.53
30–40	Zr	90.6	2.53
40–50	Sn	119.0	2.84
50–60	Nd	144.3	2.53
60–70	Yb	172.0	2.52
70–79	Au	197.2	2.80
79–92	U	238.5	3.20

The table shows that the increment 2.00 occurs twice, and 2.52 four times in the table. The increment in general increases with the atomic number.

As has been stated, if the first nine elements are considered, the average deviation of the atomic weights (O = 16) from whole numbers is only 0.019 unit, which is an extremely small deviation. If the last ten elements in Table I of the preceding paper are taken, it is found that the deviation, though much larger, is still small, and is equal to 0.075 unit. The last of these ten elements is cobalt, the second element in the eighth group for the first occurrence of the eighth group in the periodic table. Table III shows that at this point the deviation suddenly jumps to a relatively large value, being 0.32 for nickel, 0.43 for copper, and 0.37 for zinc, with an average of 0.247 for the ten elements beginning with nickel and ending with rubidium. The average deviation for the next ten elements, beginning with strontium and ending with cadmium, is also 0.247 unit, for the ten from indium to cerium it is 0.199, and for the twelve elements from tantalum to uranium, it is 0.260 unit. However, the value of Table II, as it stands, is very slight on account of the large probable errors in many of the atomic weights. This can be remedied by the choice of only such elements from the table as have accurately determined atomic weights. If thirteen elements are thus chosen as follows: nickel, copper, zinc, arsenic, bromine, rubidium, strontium, rhodium, silver, cadmium, iodine, caesium, and barium, the average deviation is 0.248 unit, while the theoretical deviation calculated on the basis that the atomic weights show no tendency to be near whole or any other special numbers, is 0.250 unit. Therefore, the tendency for the atomic weights to approximate whole numbers, which is very marked for the elements from helium up to an atomic weight of 59 (cobalt), seems to altogether disappear at the atomic weight 59 (beginning with nickel) and is not found for any of the elements which have an atomic weight higher than this value.

The reason for this abrupt change at the atomic weight 59, is not apparent. It may be in some unknown way connected with the first ap-

TABLE III.—DEVIATIONS OF THE ATOMIC WEIGHTS FROM WHOLE NUMBERS, SHOWING THAT FOR THE HEAVIER ELEMENTS THERE IS NO TENDENCY FOR THESE WEIGHTS TO APPROXIMATE WHOLE NUMBERS.

Heavier elements.								Lighter elements.[1]		
Element.	At. wt.	Diff. from whole no.	Probable error in at. wt.	Element.	At. wt.	Diff. from whole no.	Probable error in at. wt.	Element.	At. wt.	Diff. from whole no.
Ni....	58.68	0.32	0.02	In....	114.8	0.2	0.5	He....	4.00	0.00
Cu...	63.57	0.43	0.05	Sn....	119.0	0.0	0.5	Li.....	6.94	0.06
Zn....	65.37	0.37	0.05	Sb....	120.2	0.2	0.3	Be....	9.1	0.10
Ga...	69.9	0.10	0.5	Te....	127.5	0.5	0.2	B.....	11.0	0.00
Ge...	72.5	0.50	0.5	I.....	126.92	0.08	0.03	C.....	12.00	0.00
As....	74.96	0.04	0.05	Xe....	130.2	0.2	0.2	N.....	14.01	0.01
Se....	79.2	0.20	0.1	Cs....	132.81	0.19	0.05	F.....	19.00	0.00
Br....	79.92	0.08	0.1	Ba....	137.37	0.37	0.03			
Kr...	82.92	0.08	0.1	La....	139.0	0.0	0.3	Av. variation,		0.024
Rb...	85.45	0.45	0.05	Ce....	140.25	0.25	0.1			
								Na....	23.00	0.00
Av. variation,		0.247		Av. variation,		0.199		Al....	27.10	0.10
								P.....	31.02	0.02
Sr....	87.63	0.37	0.03	Ta....	181.5	0.5	1.0	S.....	32.07	0.07
Y....	89.0	0.0	0.2	W....	184.0	0.0	0.5			
Zr....	90.6	0.4	0.2	Os....	190.9	0.1	0.4	Av. variation,		0.047
Cb...	93.5	0.5	....	Ir.....	193.1	0.1	0.2			
Mo...	96.0	0.0	0.1	Pt....	195.2	0.2	0.1	Ar....	39.88	0.12
Ru...	101.7	0.3	0.1	Au....	197.2	0.2	0.1	K.....	39.10	0.10
Rh...	102.9	0.1	0.05	Hg....	200.6	0.6	0.4	Ca....	40.07	0.07
Pd...	106.7	0.3	0.1	Tl....	204.0	0.0	0.2	Ti.....	48.10	0.10
Ag...	107.88	0.12	0.02	Pb....	207.1	0.1	0.1	V.....	51.00	0.00
Cd...	112.4	0.4	0.03	Ra....	226.4	0.4	0.3	Cr....	52.00	0.00
				Th....	232.4	0.4	0.5	Mn...	54.93	0.07
Av. variation,		0.247		U.....	238.5	0.5	0.5	Fe....	55.84	0.16
								Co....	58.97	0.03
				Av. variation,		0.260				
								Av. variation,		0.072

[1] For a complete table of the lighter elements see Table II of Part II.

pearance at this point of new series, possibly formed by disintegration instead of aggregation; to a change in the effect of packing, or, if atoms exist which are lighter than hydrogen, it might possibly be due to their inclusion. If the first suggestion is considered, it is found that when the elements of high atomic weight are reached several series are known to exist. Thus the isotopes of lead, lead from radium and radium B differ in atomic weight by eight units, the isotopes radium F and radium A differ by the same amount, and radio-thorium and uranium X_1 differ by six units. If the members of the actinium series could be included, some of these differences in the weights of one species of atom would be made even larger. Where such differences exist in the weights of the different atoms having a single atomic number, it cannot be ex-

pected that any very simple relations can be found to exist for atoms of a high atomic weight, except where it is possible to compare the weights of the members of a single series, such as the uranium-radium, the thorium, or the actinium radioactive series.

It is quite possible that these differences of series go downward in the periodic system to relatively low atomic weights. Thus Aston claims to have separated neon, with an atomic weight 20.2, into neon and meta-neon, the atomic weights for which have been found by Thomson to be 20 and 22, so that the deviation of neon from the law of the approximate whole number by the amount +0.2 is probably only an apparent one. It is of interest that the difference between the atomic weights of neon and meta-neon, as found by Thomson, is two, which is the same as the average increment in the weights of the lighter elements, and is equal to the average difference between the weights of isotopes in the radioactive series. This average difference has been supposed to be also the actual difference between any two adjacent isotopes as listed below under any single atomic number:

Atomic number.	
82.	Lead from Ra, Lead from Th, Ra D, Th B, Ra B.
83.	Bi, Ra E, Th C, Ra C.
84.	Ra F, Th C, Ra C, Th A, Ra A.
86.	Th Em, Ra Em (Nt).
88.	Th X, Ra, Ms Th.
90.	Ra Th, Io, Th, UX_1.

However, these assumed differences of two have depended upon the fact that the atomic weights used for uranium and thorium have been 238.5 and 232.4, or a difference of practically 4 plus 2. The latest determination of the atomic weight of uranium by Hönigschmidt[1] gives a value of 238.18, which would not accord with this relationship for the individual differences. The difference between two isotopes belonging to a single radioactive series is, however, not affected by this result, and may still be assumed as four. However, in radioactive changes where a helium atom is lost, the new atom which is formed is not exactly four units lighter than the parent atom, since the packing effect varies with the change. How this effect varies in these heavy atoms cannot be told from the data now available, since the accuracy of the atomic weight determinations is not sufficient for this purpose, but the variation may be calculated approximately from the heat evolved in all cases where the heat change can be determined. It is of course self-evident that for deductions in regard to such atomic weight relations, the percentage accuracy must be much greater than is necessary for the study of the lighter elements. The difference between Hönigschmidt's values for uranium and for radium[2] (at.

[1] *Z. Electrochem.*, **20**, 449 (1914).

[2] *Sitzungsb. kais. Akad. Wien.*, **121**, *Abt. IIA*, 1973 (1912); *Monatsh.*, **34**, 283 (1913).

wt. = 225.97) is 12.21, or 0.21 more than the weight of three helium atoms.

Now that certain elements have been found to exist in isotopic forms, it becomes apparent that still other elements may do the same in cases which have not been recognized, so that in dealing with any single species of element it is uncertain whether this is an individual with respect to its atomic weight. The great regularity with which the elements follow the relationships given in these papers, up to an atomic weight of 59, suggests that with the exception of the cases of neon, silicon, magnesium, and chlorine, isotopes probably do not exist to any large extent for any of these elements, if they exist at all. There is still another possibility which suggests itself, and that is that the different atoms of a single atomic species differ in weight among themselves, and that the atomic weights as found are simply statistical averages. If this were true, the constancy of the results obtained in atomic weight determinations which after all is not of an extremely high order, would be due to the fact that in a single determination such an enormous number of atoms is used. For example, if in one determination the weight of silver chloride obtained were 7.16 g., the number of chlorine or silver atoms in the precipitate would be 3×10^{22}, or thirty thousand billion billion. The statement of the above idea is not meant to be understood as an advocacy of such a theory, but only to point out the possibility that such might be the case.

TABLE IV.—A SYMBOLICAL REPRESENTATION OF THE ATOMIC WEIGHTS OF THE ELEMENTS IN THE FIRST THREE SERIES OF THE PERIODIC TABLE.

H = 1.0078.

	0.	1.	2.	3.	4.	5.	6.	7.	8.	
Ser. 2.	He He	Li He+H_3	Be 2He+H	B 2He+H_3	C 3He	N 3He+H_2	O 4He	F 4He+H_3		
Theor.	4.00	7.00	9.0	11.0	12.00	14.00	16.00	19.00		
Det..	4.00	6.94	9.1	11.0	12.00	14.01	16.00	19.00		
Ser. 3.	Ne 5He	Na 5He+H_3	Mg 6He	Al 6He+H_3	Si 7He	P 7He+H_3	S 8He	Cl 8He+H_3		
Theor.	20.0	23.00	24.00	27.0	28.0	31.00	32.00	35.00		
Det..	20.0	23.00	24.32	27.1	28.3	31.02	32.07	35.46		
Ser. 4.	A 10He	K 9He+H_3	Ca 10He	Sc 11He	Ti 12He	V 12He +H_3	Cr 13He	Mn 13He +H_3	Fe 14He	Co 14He +H_3
Theor.	40.0	39.00	40.00	44.0	48.0	51.0	52.0	55.00	56.00	59.00
Det..	39.9	39.10	40.07	44.1	48.1	51.0	52.0	54.93	55.84	58.97

Increment from Series 2 to Series 3 = 4He
Increment from Series 3 to Series 4 = 5He (4He for K and Ca)
Increment from Series 4 to Series 5 = 6He

Table IV gives Series 2, 3 and 4 of the periodic system, built up by adding the weight of one helium atom for each change of two places to the right, and by adding enough multiples of the weight of a hydrogen atom to make up the atomic weight. In order to make the relationship ap-

parent a symbolical representation has been used, He being taken to stand for the weight 4, and H for the weight 1.00. Built up in this way, the atomic weights of all of the members of the even numbered groups (with the exception of beryllium) may be represented by a whole number of symbols He, while all of the atomic weights in the odd groups may be represented by 3H plus a whole number of symbols He.

In the fourth, or argon series, the atomic weights begin to increase more rapidly than in the second and third series. This effect is first seen in the case of argon, which with a calculated atomic weight of 36, has instead a weight of practically forty, or too much by the weight of one helium atom. This effect dies out in potassium and calcium, and then appears again in scandium, titanium and the other members of this series. It becomes apparent in another way on studying the increment of weight in passing from a member of one series to the corresponding member of the series below it. Thus the second member in each group is obtained from the first by adding 4He. In going from the second to the third member of the group the increase is the same (4He) to give potassium or calcium, but is 5He to give argon, titanium, vanadium, chromium, and manganese.[1] This in a sense explains how the atomic weight of argon comes to be greater than that of potassium, and practically equal to that of calcium. In going from the third to the fourth member of each group, it is necessary to add 6He, but the increase in this case seems to be due to the interposition of the eighth group elements, iron, cobalt, and nickel.

While both the law of the approximate whole number, and the hydrogen–helium system here presented, become suddenly much less accurate beginning with the element nickel, this does not necessarily mean that the hydrogen–helium system breaks down at this point, since there are several possible causes, already mentioned, which may account for the sudden increase in the deviations. The eighth group fills the position of a transition group between the seventh group and the first, which shows that it fills exactly the place of the zero group in the other series. The first member of the eighth group tried thus has an even number as its atomic number. The second member has an odd, and the third an even number, which gives to the first group an odd atomic number. This is entirely in accord with the system, which would fail at this point if there had been two instead of three members in each position in the eighth group.

According to the rule that the atomic weights of the elements increase alternately by 3 and by 1, then since iron has a weight of 56, that of cobalt should be 59 (detd. = 58.97), nickel should be 60 (detd. = 58.68), and

[1] In comparison with the other members of the same series it is potassium and calcium rather than argon, which are exceptional.

copper 63 (detd. = 63.57). The first large negative deviation among the elements of even atomic numbers, of any of the actual atomic weights from the theoretical value, is thus found for the element nickel. Now it has been found that if it is studied from the standpoint of its behavior toward X-rays, nickel behaves as an element of a considerably higher atomic weight than the determined value. The wave lengths of the strong K radiations as found by Moseley are proportional to the reciprocals of the squares of the atomic weights. If cobalt is taken as a standard of reference (the square of the atomic weight and wave length being taken as 100 for this element), the values, part of which were calculated by Kaye,[1] come out as follows:[2]

	Al.	Si.	Cl.	K.	Ca.	Ti.	V.	Cr.	Mn.
(Atomic weight)	21.1	23.0	36.1	44	46	66	75	78	86
1/Wave length	21.5	25.2	37.8	47	53	65	72	78	85

	Fe.	Co.	Ni.	Cu.	Zn.	Rh.	Pd.	Ag.
(Atomic weight)	90	100	99	116	123	304	328	334
1/Wave length	92	100	108	116	124	298	314	321

The atomic weight of nickel, if calculated from the value 108 as given in this table, comes out as about 61.2, while the other elements from titanium up to and including rhodium, give a very close agreement. The principle as given above is derived from Whiddington's result that the energy of a characteristic X-ray is roughly proportional to the atomic weight, and from the quantum theory of radiation, according to which the energy of a radiation is inversely proportional to its wave length.

A study of the packing effects, as given in Table II of the preceding paper, shows that where an atom is built up entirely of helium atoms, then, on the average, the decrease in mass is practically due entirely to the primary formation of the helium atoms, and not at all to the aggregation of these into atoms which are heavier. From this point of view an atom composed entirely of helium units would have extreme instability in so far as its disintegration into helium units, in *comparison* with its instability with reference to a hydrogen decomposition. Such an atom in a radioactive transformation should lose α-particles much more readily than hydrogen nuclei, in fact, if it is remembered that the alpha decomposition is itself not complete in any case, it will be seen that it is doubtful if such an atom would ever give a detectable hydrogen disintegration.

If the atoms are built up entirely according to the special system presented in Table IV, according to which the members of even numbered groups are in general aggregates of helium alone, then since all of the radioactive elements which are now known to give a simple alpha decom-

[1] "X-Rays," 200.

[2] This table could be extended by including the values of the nuclear charge, when it would be seen that the wave lengths seem to be determined by the nuclear charge as found by Moseley, rather than by the atomic weight.

position (that is without an accompanying beta change) belong to even numbered groups, they could not be expected to give hydrogen upon disintegration. Thus one of the chief objections to the theory that the atoms are hydrogen complexes, which is based on the fact that up to the present time no hydrogen has been detected as the product of any radioactive change, is seen to be not contrary to, but rather in accord with, the theory as presented in these papers. The exceptional case of beryllium shows, however, that even numbers of even numbered groups sometimes contain a hydrogen nucleus which was not contained in one of the helium nuclei from which the atom was built, so that there still remains the possibility, though the probability seems small, that hydrogen nuclei might be liberated from atoms belonging to these groups. There is no evidence that the particular system presented in Table IV holds exactly for the atoms of high atomic weight, but the general form of the system indicates at least that the atoms contain more helium than independent hydrogen units, and this seems in accord with the fact that uranium loses α-particles in eight steps, and is changed into a form of lead, without any apparent loss of a hydrogen nucleus.

The stability with which the hydrogen nuclei which are not contained in helium groups, but which generally occur in threes (H_3 in Table II), are built into the complex atoms, is not indicated with any degree of accuracy, but in the case of lithium it seems to be great, for lithium shows the extremely large packing effect equal to 1.57%, which might seem doubtful but for the care taken by Richards and Willard[1] in the determination of this atomic weight.

The hydrogen–helium system here presented is entirely in accord with, but independent of, the astronomical theory that the order in which the elements appear in the stars is first nebulium, hydrogen and helium, then such of the lighter elements as calcium, magnesium, oxygen, and nitrogen, and finally iron, and the other heavy metals, although in the present system it has not been found necessary to include nebulium. Some of the nebulae give bright line spectra of nebulium, hydrogen and helium, such Orion stars as those of the Trapezium give lines for hydrogen and helium, while those that are more developed show magnesium, silicon, oxygen and nitrogen, and some of the other low atomic weight elements in addition. Bluish white stars such as Sirius give narrow and faint lines for iron, sodium, and magnesium, and the solar stars give much weaker hydrogen spectrum, and many more and stronger lines for iron and the heavy metals. The astronomical theory that the heavier elements are thus formed from those of smaller atomic weight is of extreme interest, but the evidence for it is somewhat uncertain, since it is possible that it is the difference in the density of the different elements which

[1] Richards and Willard, *J. Am. Chem. Soc.*, **32**, 4 (1910).

is the effective factor in causing the spectra to appear in the order in which they are found to occur. The relative brightness of the different lines also varies greatly, such lines as the calcium H and K lines being extremely strong, and this also interferes with the determination of the order of the appearance of the elements in the stars. On the other hand, the evidence presented in these papers, which seems to show that the elements are atomic compounds of hydrogen and helium, appears to give some support to the theory of the evolution of the heavier atoms from those which are lighter. The evidence for the hydrogen–helium system is, however, very much stronger and more complete than that for the evolution of the elements in the stars.

Summary.

1. The fundamental idea of this paper on atomic structure, is to show that the system which has been found to apply to the atomic weight and valence relations of the members of each of the radioactive series, also holds true for the lighter atoms. In a radioactive series it is found that a loss of an α-particle with a mass of four decreases the valence by two, and thus shifts the element two groups to the left in the periodic table, and decreases the atomic number by two. If this is true for the lighter elements, beginning with helium, then the addition of the weight of a helium atom for each increase of two in the atomic number ought to give the atomic weights of the elements belonging to the even numbered groups. The atomic weights found by this method are the same on the whole as the determined values, which shows that the theory accords with the facts.

2. The lithium atom, which is the first atom in the odd numbered group, is heavier than the helium atom by the weight of three hydrogen atoms. It would be very remarkable if the atoms of odd atomic number follow the same rule as those of even atomic number, but that they do is indicated by Table IV, which shows that for the odd numbered groups as well, each increase of two in the atomic number results in an increase of four in the atomic weight.

3. The atomic weights of the lighter elements are given with considerable accuracy by the equation

$$W = 2n + (1/2 + 1/2\,(-1)^{n-1}),$$

where W is the atomic weight and n the atomic number. In the case of the heavier elements another term enters, so that the more general equation may be given:

$$W = 2\,(n + n') + [1/2 + 1/2\,(-1)^{n-1}]$$

4. Of the 27 elements from helium to cobalt, 13, or nearly one-half, have atomic weights divisible by four, and these elements in general belong to even numbered groups in the periodic table. Of all the possi-

ble multiples of four only two are missing, *i. e.*, 2×4 and 9×4, and seemingly to make up for the omission of the 9×4, the 10×4 occurs twice. An explanation of the omission of the 2×4 and its occurrence as $(2 \times 4) + 1$ will be given in a later paper.

5. If the atomic weights increase by the weight of one helium atom for an increase of two in the atomic number, the average increase in the atomic weight per atomic number should be 2. That this is in accord with the facts is shown, for neon with an atomic number 10 has an atomic weight of 10×2 or 20, and calcium, with an atomic number 20, has an atomic weight equal to 20×2 or 40.

6. According to Part II, the magnitude of the packing effect for helium is 0.77%, which is the same as the average of the packing effects for the first 27 elements, so that if a more complex atom is built of helium groups alone, then in general nearly all of the packing effect is due to the primary formation of the helium nucleus from four hydrogen nuclei and two negative electrons, and almost no packing effect results from the aggregation of these helium nuclei into more complex atoms. On this view the helium nuclei must be very greatly more stable than the nuclei of the more complex atoms which they form, so that such an atom, made up entirely from helium units, should give helium and not hydrogen by its primary decomposition. This is in accord with the behavior of the radioactive elements when they disintegrate. It is of interest to note that the members of the radioactive series which are now known to give helium on decomposition, belong to the even numbered groups on the periodic table, and therefore to those groups which are shown in Table IV, as helium aggregates alone. That these heavy atoms must contain a considerable number of helium units is shown by the fact that uranium changes into lead by eight steps in which it loses α-particles.

7. The hydrogen–helium system gives an explanation of the fact that argon has an atomic weight of 40, which is higher than that of potassium, which has an atomic number higher by 1. A study of Table IV makes the reason apparent, and shows that in *comparison* with other members of Series 4 in the periodic table, it is potassium and calcium, and not argon, which are exceptional. In comparison with the members of Series 3, and potassium and calcium, it is of course the argon which is exceptional. As the atoms grow heavier there is a tendency to take on helium (or perhaps hydrogen) groups more rapidly than is the rule in the case of the lighter elements.

The writer wishes to express his sincere thanks to Dr. W. D. Harkins, under whom this investigation has been carried out, for his continued interest in the work, and for his many suggestions.

CHICAGO, ILL.

MAY 20 1920
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